MUonE: extraction of $\Delta\alpha_{had}$

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Workshop on Muon Precision Physics, Liverpool, 7-9 November 2022

https://indico.ph.liv.ac.uk/event/731/

References

MUonE analysis first described in the LoI
Letter-Of-Intent SPSC-I-252 (June 2019)

Updated description in a recent paper:
Phys. Scripta 97 (2022) 054007 [arXiv:2201.13177]

Latest advancements: Riccardo Pilato's PhD thesis

Web pages with links to documents (papers, conferences, theses)

- https://web.infn.it/MUonE/

Analysis workflow: FastSim - FullSim

A competitive determination of a_{μ}^{HVP} requires a precision of $O(10^{-2})$ in the measurement of the hadronic running, which translates into an unprecedented precision of $O(10^{-5})$ in the shape of the differential cross section.

Reaching this accuracy requires a huge statistics of data, in the order of few times 10^{12} events.

Therefore even preliminary simulation studies would present a computational challenge.

The Fast Simulation is an absolute need for MUonE: the final analysis will likely use Fast Simulation, with detector effects parameterized from (smaller and dedicated) Full Sim samples.

Therefore the FastSim development has to proceed in parallel with the Full Simulation.

Given the above, the MC Data Formats representing MUonE MC events should be common to Fast and Full Simulation, such that analyses could be carried out in a similar way, with the same interfaces.

MuE: FastSim – Analysis code

Public code: https://gitlab.cern.ch/muesli/nlo-mc/mue

- Package have been existing for quite some time, used in:
  - LoI analysis and updates
  - Studies of systematics (G.A., R.Pilato, M.Mantovano)
  - Fast Simulation of the TestRun setup, the Beam Profile and the ECAL (E.Spedicato, S.Cesare)
  - Starting point for the study of the simultaneous fit of signal+nuisances with Combine (R.Pilato)

- Recent updates:
  - use NNLO MESMER MC generator for μe scattering
  - embedded mode for MESMER MC generator, with (Fortran) generator functions called from a C++ steering program.

FastSim analysis strategy

- NLO MESMER MC
- $\Delta\alpha_{had}(t)$ from F.Jegerlehner's code(hadr5n12.f)→ $a_\mu^{HLO} = 688.6 \times 10^{-10}$
- Detector resolution effects parametrized in a simplified way (including only: multiple scattering on 1.5cm Be target and intrinsic resolution σ_θ=0.02 mrad)
  - Neglecting: scattering on the Si planes, non-Gaussian tails, residual backgrounds
  - Neglecting: detailed track simulation and reconstruction
- Fit is done directly on the angular distributions of scattered μ and e
  - No attempt to estimate t (or x) event by event
  - $\theta_e < 32$ mrad (geometric acceptance)
  - $\theta_\mu > 0.2$ mrad (remove most of the background)
  - Both 1D and 2D distributions fitted. 2D is the most robust.
  - Ideally there is no need to identify the outgoing muon and electron, provided the event is a signal one. In this case we simply label the two angles as θ_L, θ_R ("Left" and "Right" w.r.t. an arbitrary axis)
- Shape-only fit: the absolute normalization shall not count.

Hadronic running of α

Most easily displayed by taking ratios of the MC predicted angular distributions (pseudodata) and the predictions obtained from the same MC sample reweighting $\alpha(t)$ to correspond to only the leptonic running.

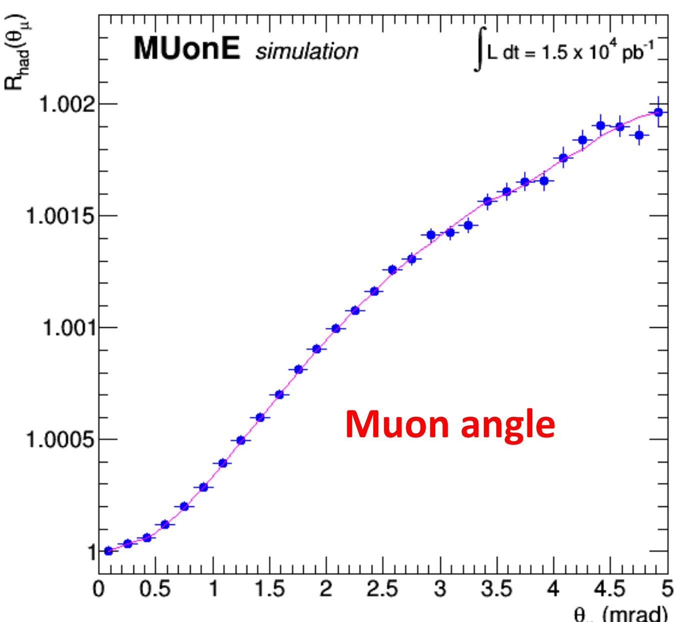
$$R_{had}(\theta) = \frac{d\sigma(\theta, \Delta\alpha_{had})}{d\sigma(\theta, \Delta\alpha_{had}=0)}$$

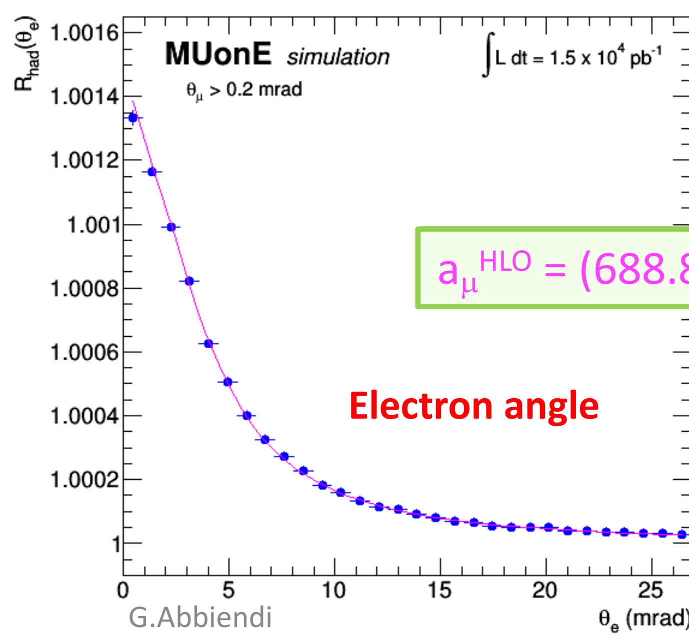
-- In this way most of the pure MC statistical fluctuations are cancelled.

-- (of course, this trick is applicable only to pseudodata analysis. With real data we will need to match the MC statistics to the data size)

Observable effect ~ 10^{-3} / wanted precision ~10^{-2} → required precision ~10^{-5}

The expected distributions are obtained from the nominal integrated luminosity
$L = 1.5 \times 10^{7}\ nb^{-1}$ (corresponding to 3-year run)





Example toy experiment

$a_\mu^{HLO} = (688.8 \pm 2.4) \times 10^{-10}$

Stat.err. 0.35%

$\Delta\alpha_{had}$ parameterisation

Physics-inspired from the calculable contribution of lepton-pairs and top quarks at t<0

$q^2 = t < 0$

$$\Delta\alpha_{had}(t) = k\left\{-\frac{5}{9} - \frac{4M}{3t} + \left(\frac{4M^2}{3t^2} + \frac{M}{3t} - \frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

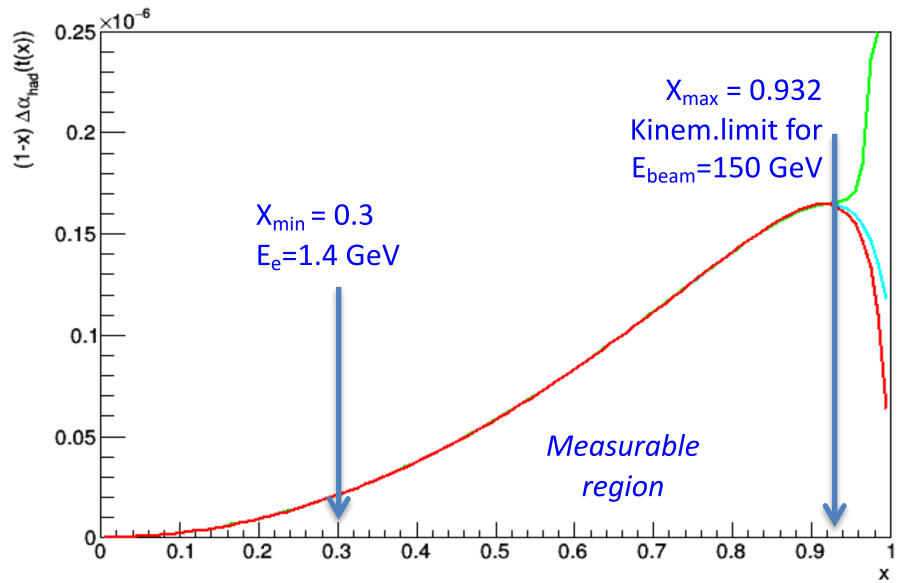
M with dimension of mass squared, related to the mass of the fermion in the vacuum polarization loop
k depending on the coupling $\alpha(0)$, the electric charge and the colour charge of the fermion

Low-|t| behavior dominant in the MUonE kinematical range:

$$\Delta\alpha_{had}(t) \simeq -\frac{1}{15}\frac{k}{M}t$$

a_μ^{HLO} calculable from the master integral in the FULL phase space with this parameterisation.

Instead simple polinomials diverge for x->1 (green is a cubic polinomial in t)



Template Fit technique

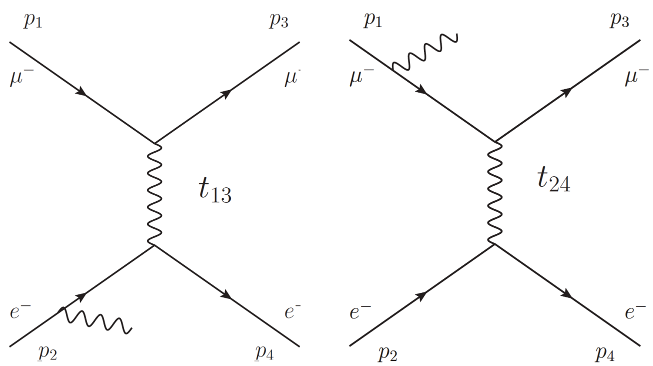
- MC templates for any useful distribution are built by reweighting the events to correspond to a given functional form of $\Delta\alpha_{had}(t)$

- $\Delta\alpha_{had}(t)$ is conveniently parameterised with the "Lepton-Like" form, one-loop QED calculation.

The 2->3 matrix element for one-photon emission at NLO can be split in 3 parts (radiation from mu or e leg and their interference), each one with a different running coupling factor → **3 coefficients**

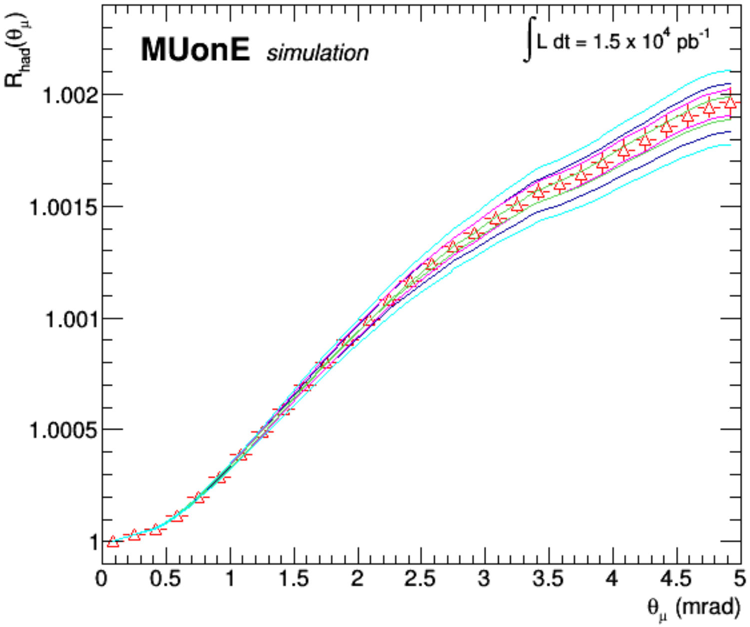
NOTE: at NNLO one needs **11 coefficients**

By saving the relevant coefficients at generation time we can easily reweight the events according to the chosen parameters in the $\Delta\alpha_{had}(t)$



Template fit

- Define a grid of points (K,M) in the parameter space covering a region of ±5σ around the expected values (with σ the expected uncertainty). Step size taken to be 0.5σ. This defines 21x21 = 441 templates for the relevant distributions.



- For every template in the grid calculate the χ^2 obtained with the pseudodata distribution:

$$\chi^2(K,M) = \sum_i^{bins} \frac{R_i^{data} - R_i^{(K,M)}}{\sigma_i^{data}}$$

  - Neglect the statistical errors of the templates as in the ratios they are vanishingly small.

- Minimise the χ^2 interpolating across the grid by parabolic approximation. Final errors correspond to $\Delta\chi^2=1$.

a_μ^{HLO}

- From the fitted (K,M) values the hadronic contribution to $\Delta\alpha_{had}(t)$ is determined from the Lepton-Like parameterisation:

$$\Delta\alpha_{had}(t) = k\left\{-\frac{5}{9}-\frac{4M}{3t}+\left(\frac{4M^2}{3t^2}+\frac{M}{3t}-\frac{1}{6}\right)\frac{2}{\sqrt{1-\frac{4M}{t}}}\log\left|\frac{1-\sqrt{1-\frac{4M}{t}}}{1+\sqrt{1-\frac{4M}{t}}}\right|\right\}$$

- Then, by using the master integral, we have the result in the full phase space:

$$a_\mu^{HLO} = \frac{\alpha}{\pi}\int_0^1 dx(1-x)\Delta\alpha_{had}[t(x)]$$

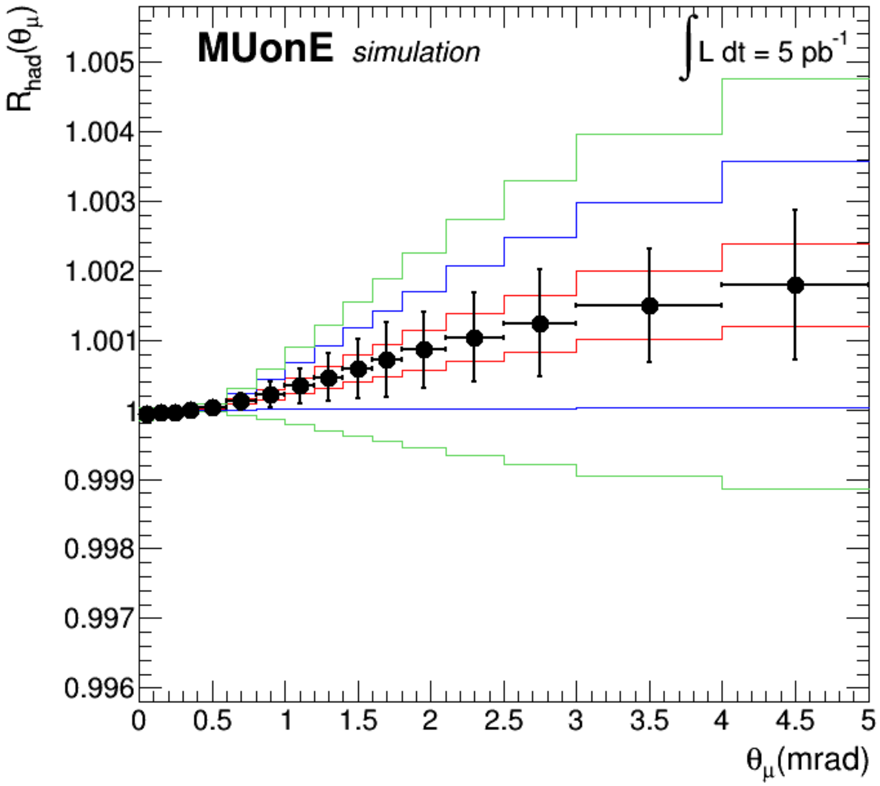
- The result for the nominal luminosity is $a_\mu^{HLO} = (688.8 \pm 2.4) \times 10^{-10}$
  - statistical uncertainty of 0.35%

- The expectation from the used Jegerlehner's parameterization is: $a_\mu^{HLO} = 688.6 \times 10^{-10}$
  - difference from our fit is 0.2×10^{-10}, negligible w.r.t. the statistical uncertainty

Expected sensitivity of a First Physics Run

Expected integrated Luminosity with the Test Run setup with full beam intensity & detector efficiency ~ $1 pb^{-1}$/day

In one week ~$5 pb^{-1}$ → ~10^9 μe scattering events with $E_e > 1$ GeV ($\theta_e < 30$ mrad)



Initial sensitivity to the hadronic running of α.

Pure statistical level: 5.2σ
2D (θ_μ,θ_e) K=0.136 ± 0.026

Definitely we will have sensitivity to the leptonic running (ten times larger)

Template fit with just one fit parameter K= k/M in the $\Delta\alpha_{had}$ parameterization.
The other parameter fixed at its expected value: M = 0.0525 GeV^2

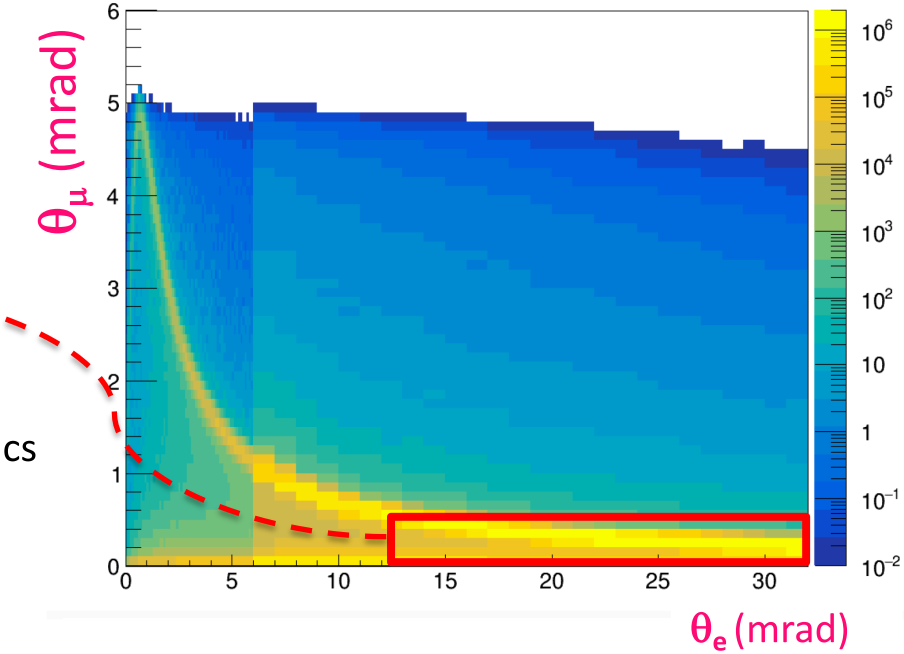
Event kinematics

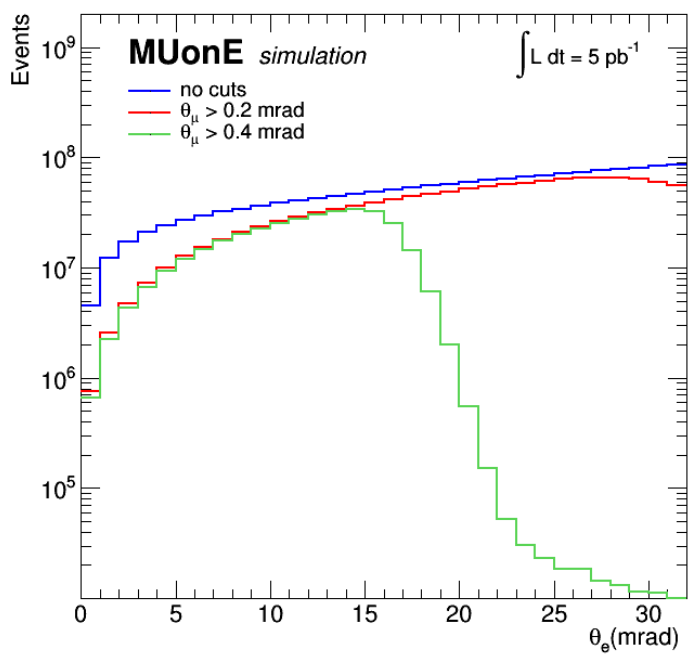
Normalisation region

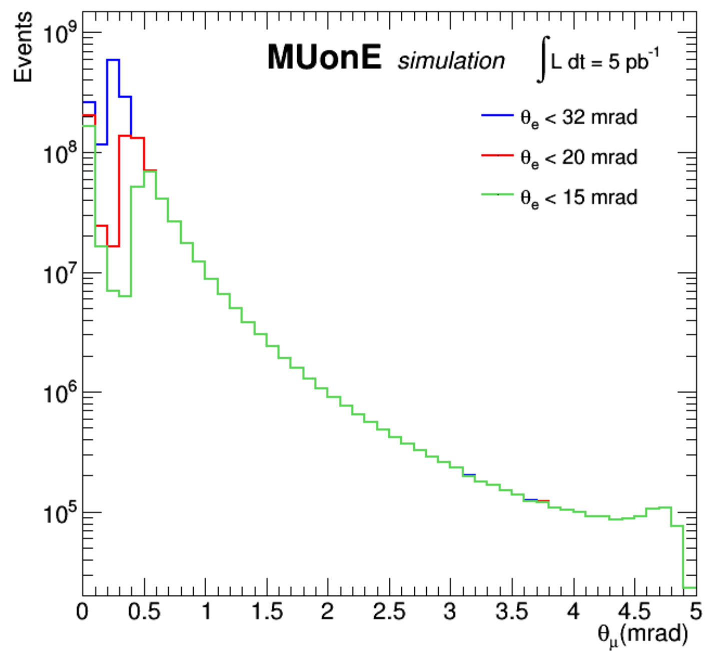
huge statistics

vanishing signal ($\Delta\alpha_{had}$)

convenient for studies of systematics







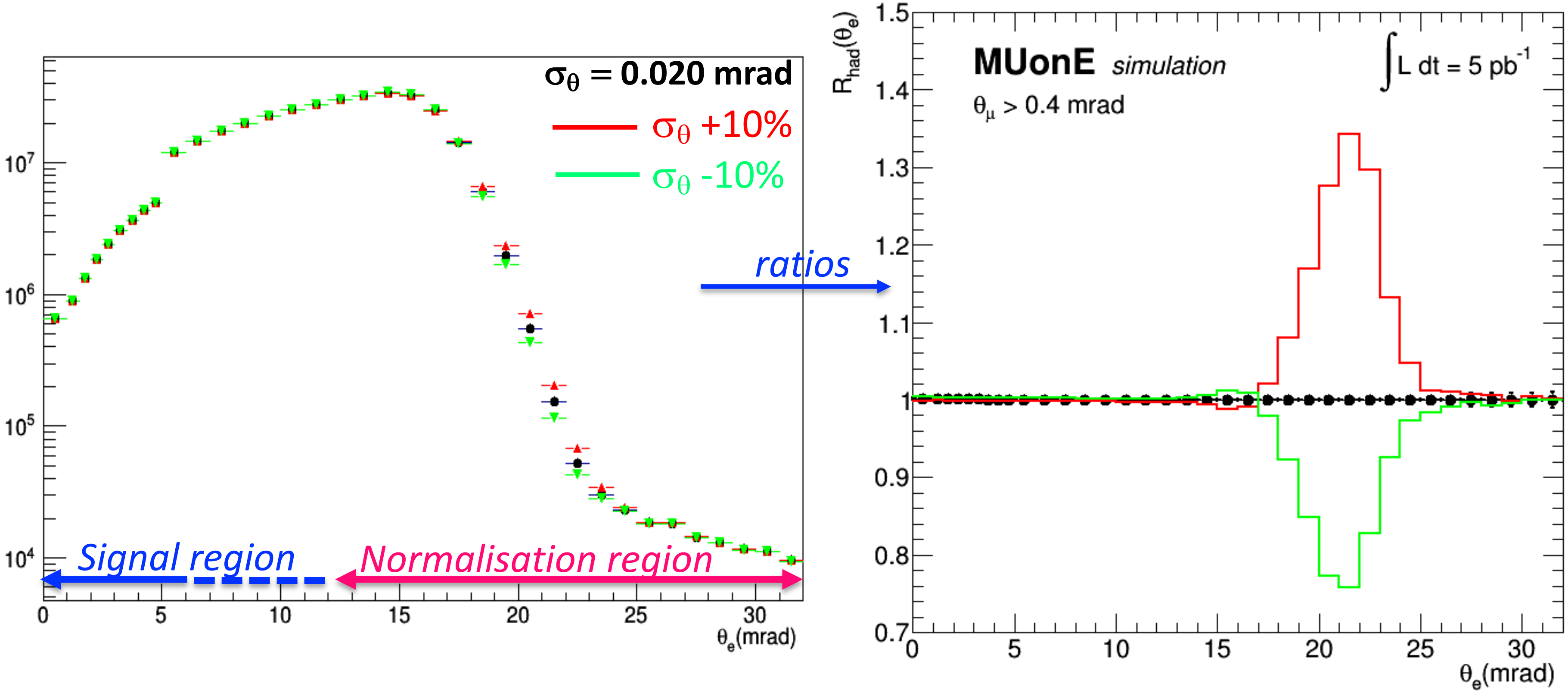
Probing systematics in the normalisation region

The **intrinsic angular resolution** can be probed by looking at the θ_e distribution after a cut on θ_μ distribution, e.g. cutting at $\theta_\mu > 0.4$ mrad

→ Effect of a ±10% error w.r.t. the nominal σ_θ = **0.020 mrad**

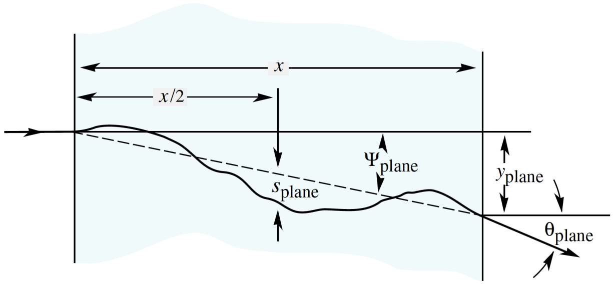
Huge distortion of 20-30% around electron angles of 20 mrad

No effect in the signal region

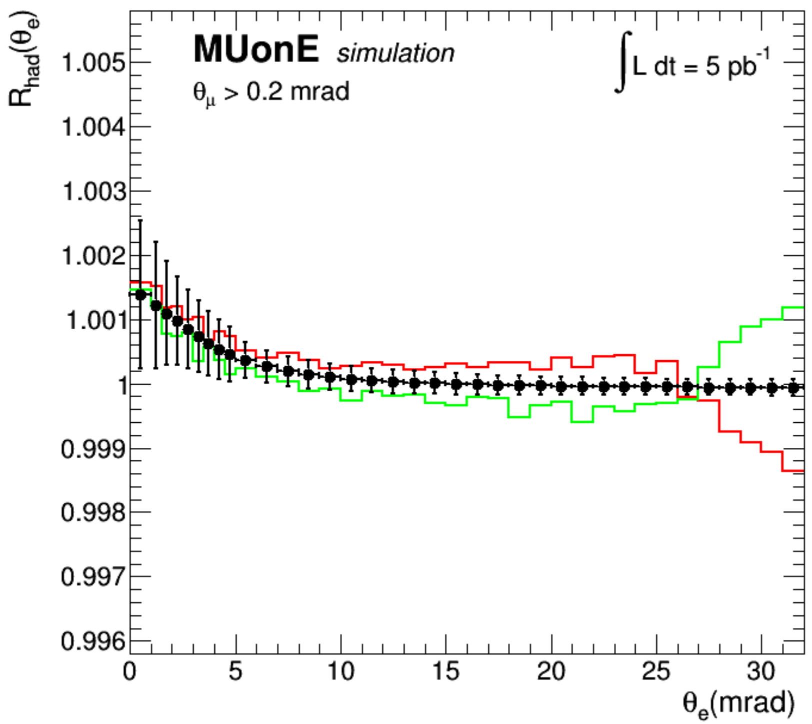


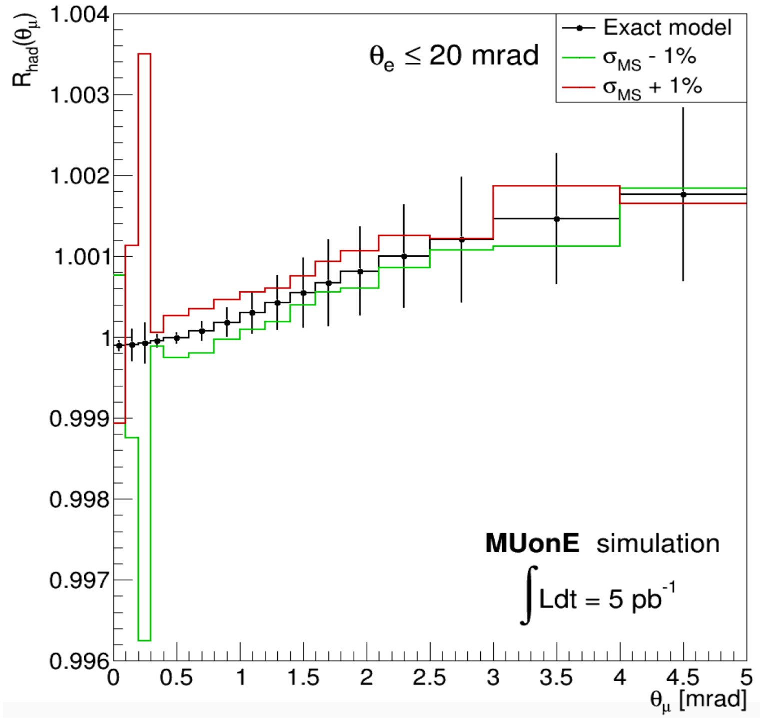
Systematics: Multiple Coulomb Scattering

Effect of a flat error of ±1% on the core width of multiple scattering



+1%
-1%





Multiple scattering previously studied in a Beam Test in 2017: [JINST 15 (2020) P01017]
with 12–20 GeV electrons on 8-20 mm C targets

Systematics: Beam Energy scale

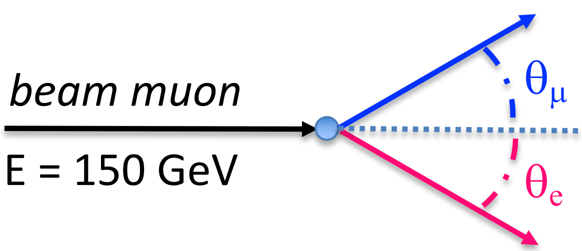
Time dependency of the beam energy profile has to be continuously monitored during the run:

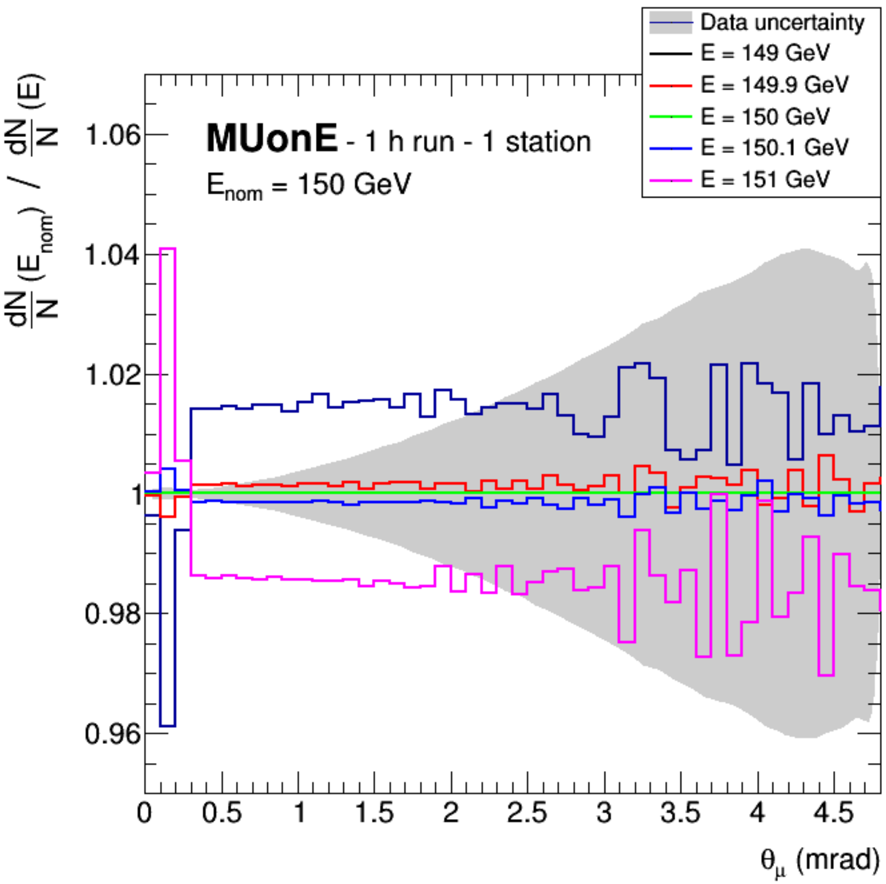
- SPS monitor
- COMPASS BMS

} needed external infos

Effect of a syst shift of the average beam energy on the θ_μ distribution: 1h run / 1 station

However, the absolute beam energy scale has to be calibrated by a physics process: kinematical method on elastic μe events





For equal angles:

$\theta_\mu = \theta_e \equiv \theta$

$$\theta \simeq \sqrt{\frac{2m_e}{E}}$$

Can reach <3 MeV uncertainty in a single station in less than one week

From SPS E scale ~1% : 1.5 GeV

Simultaneous fit of signal and nuisances

developed by Riccardo Pilato (PhD thesis) + L.Bianchini + G.A.

- Template fit using CMS Combine tool
  - Likelihood fit with systematics included as nuisance parameters, simultaneously extracted along with the signal parameters
    https://cms-analysis.github.io/HiggsAnalysis-CombinedLimit/

- Currently including 4 nuisance parameters, related to:
  - Normalisation (uncertainty in the integrated luminosity)
  - Average beam energy
  - Intrinsic angular resolution
  - Multiple Coulomb Scattering (core width)

- Recent improvement: two-step workflow suited to the Test Run luminosity
  1. fit the main systematic effects (nuisance parameters) in the normalisation region (where signal is ~0). Use a short run of ~1h time station-by-station (~35/nb) assuming SM for the hadronic running.
  2. fit signal + nuisance parameters using as starting values and prior uncertainties for the nuisances the values determined in (1). Use the full Test Run statistics (~5/pb).

Initial ansatz

Normalisation

- Expected uncertainty on the integrated luminosity $\varepsilon \simeq 1\%$ modelled as a lognormal nuisance on the bin contents:

$$n_i \to n_i(\nu) = n_i(1+\varepsilon)^\nu$$

Shape nuisances

- Beam energy scale known to ~1% from the SPS
- Expected uncertainty on the intrinsic angular resolution ~10%
- Expected uncertainty on the core width of the multiple Coulomb scattering ~1%
  - (consistent with our results in JINST 15 (2020) P01017)

Example Fit results

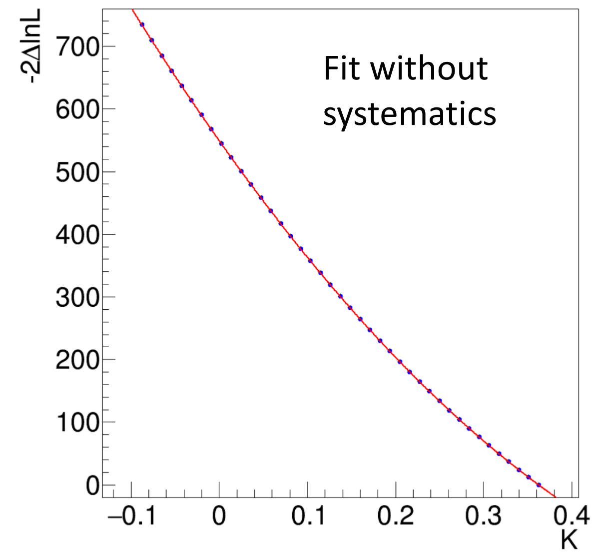
Pseudodata with statistics equivalent to the Test Run (5/pb). Input Signal parameter: K=0.136. Normalisation nuisance ν=0. Distorted by the following 3 simultaneous shape systematics:

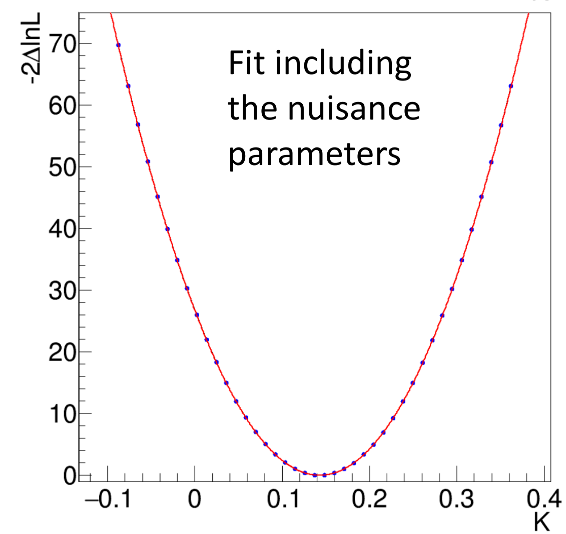
	Nominal configuration	Shift in the pseudo-data
Beam energy scale	$E_{beam} = (150 \pm 1)$ GeV	+ 6 MeV
Multiple scattering	$\sqrt{x/X_0} = 0.1458 \pm 1\%$	+ 0.5%
Angular intrinsic resolution	$\sigma_{Intr} = 0.02\,\text{mrad} \pm 10\%$	+ 5%

Results:

Selection cuts	Fit results	
$\theta_e \leq 32\,\text{mrad}$ $\theta_\mu \geq 0.2\,\text{mrad}$	$K = 0.133 \pm 0.028$	$\mu_{MS} = (0.47 \pm 0.03)\%$ $\mu_{Intr} = (5.02 \pm 0.02)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.5)$ MeV $\nu = -0.001 \pm 0.003$
$\theta_e \leq 20\,\text{mrad}$ $\theta_\mu \geq 0.4\,\text{mrad}$	$K = 0.133 \pm 0.033$	$\mu_{MS} = (0.46 \pm 0.04)\%$ $\mu_{Intr} = (5.02 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.6)$ MeV $\nu = -0.008 \pm 0.007$
$\theta_e \leq 32\text{mrad}$ $\theta_\mu \geq 0.4\,\text{mrad}$	$K = 0.133 \pm 0.033$	$\mu_{MS} = (0.46 \pm 0.04)\%$ $\mu_{Intr} = (5.03 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.6)$ MeV $\nu = -0.009 \pm 0.007$
$\theta_e \leq 20\,\text{mrad}$ $\theta_\mu \geq 0.2\,\text{mrad}$	$K = 0.133 \pm 0.031$	$\mu_{MS} = (0.47 \pm 0.03)\%$ $\mu_{Intr} = (5.02 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.5 \pm 0.5)$ MeV $\nu = -0.001 \pm 0.006$
$\theta_{L,R} \in [0.2, 32]$ mrad	$K = 0.132 \pm 0.029$	$\mu_{MS} = (0.45 \pm 0.02)\%$ $\mu_{Intr} = (5.04 \pm 0.02)\%$ $\mu_{E_{Beam}} = (6.9 \pm 0.5)$ MeV $\nu = -0.001 \pm 0.003$
$\theta_{L,R} \in [0.4, 20]$ mrad	$K = 0.133 \pm 0.034$	$\mu_{MS} = (0.43 \pm 0.03)\%$ $\mu_{Intr} = (5.05 \pm 0.03)\%$ $\mu_{E_{Beam}} = (6.8 \pm 0.6)$ MeV $\nu = -0.008 \pm 0.007$

Fit Results are in excellent agreement with the input values for all the selections, both with and without particle identification





Conclusions

- Analysis strategy based on a template fit using
  - the best MC calculations for the μe scattering
  - a convenient parameterisation of the $\Delta\alpha_{had}(t)$
    - the so-called lepton-like parameterisation, with 2 free parameters seems optimal
  - Extrapolation to the full phase space of the fitted parameterisation
- Expected (statistical) precision:
  - 0.35% (nominal luminosity 15/fb)
  - ~20% for the Test Run with 2 stations (L~5/pb)
- Many systematics can be determined in the normalisation region where the signal is vanishing
- Residual systematics can be fitted simultaneously with the signal as nuisance parameters (with the Combine statistical tool)
- Lot of work is still to be done:
  - much has to be learnt from beam tests and the Test Run
  - much has to be done to develop final software and computing workflows

BACKUP

Backgrounds

- MUonE will observe interactions in matter, not in vacuum
  - Signal simulated by MESMER at NNLO
  - Backgrounds in μe interactions also simulated by MESMER: photon bremsstrahlung and e+e- pair production
  - Main backgrounds from μN interactions only simulated by GEANT: $\mu N \rightarrow \mu e^+ e^- N$ and $\mu N \rightarrow \mu \gamma N$

Needed work:

- MC studies of the backgrounds with the Test Run setup
- standalone MC generators for the main backgrounds to include them in Fast Sim analysis

Event Identification / Selection

- ECAL crucial for many tasks
  - e-μ identification at θ<5mrad
  - Verify the track-based selection on the last station
    - Calo-based elastic selection
  - Important handle in studies of systematics

POSSIBLE FUTURE

- Incoming Muon Spectrometer (now the BMS is an external detector)
  - Important for monitoring the beam energy scale (BMS has a limited DAQ rate)
  - Improved resolution would reduce the systematic uncertainty on the beam energy scale
  - Associated with a spectrometer for the outgoing muon it would determine precisely the event kinematics

- Muon Detector still undefined
  - It would assure 100% identification of the scattered muon (also for the full-size 40-stations detector), hence completing the ID for the events where the scattered electron cannot be reconstructed in ECAL
  - If instrumented with a magnetic field it would allow closing the kinematics for the most interesting events, giving further tools for the searches for dark matter

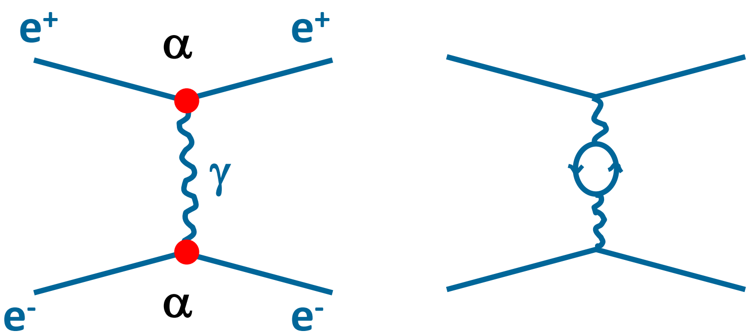
Measurement of $\Delta\alpha_{had}(t)$ spacelike at LEP

Eur.Phys.J.C45(2006)1

OPAL measurement: Bhabha scattering at small angle, with $1.8 < -t < 6.1$ GeV^2

about 10^7 events

precision at the per mille level



$$\frac{d\sigma}{dt} = \frac{d\sigma^{(0)}}{dt}\left[\frac{\alpha(t)}{\alpha_0}\right]^2 (1+\varepsilon)(1+\delta_\gamma) + \delta_Z$$

Born term for t-channel single γ exchange

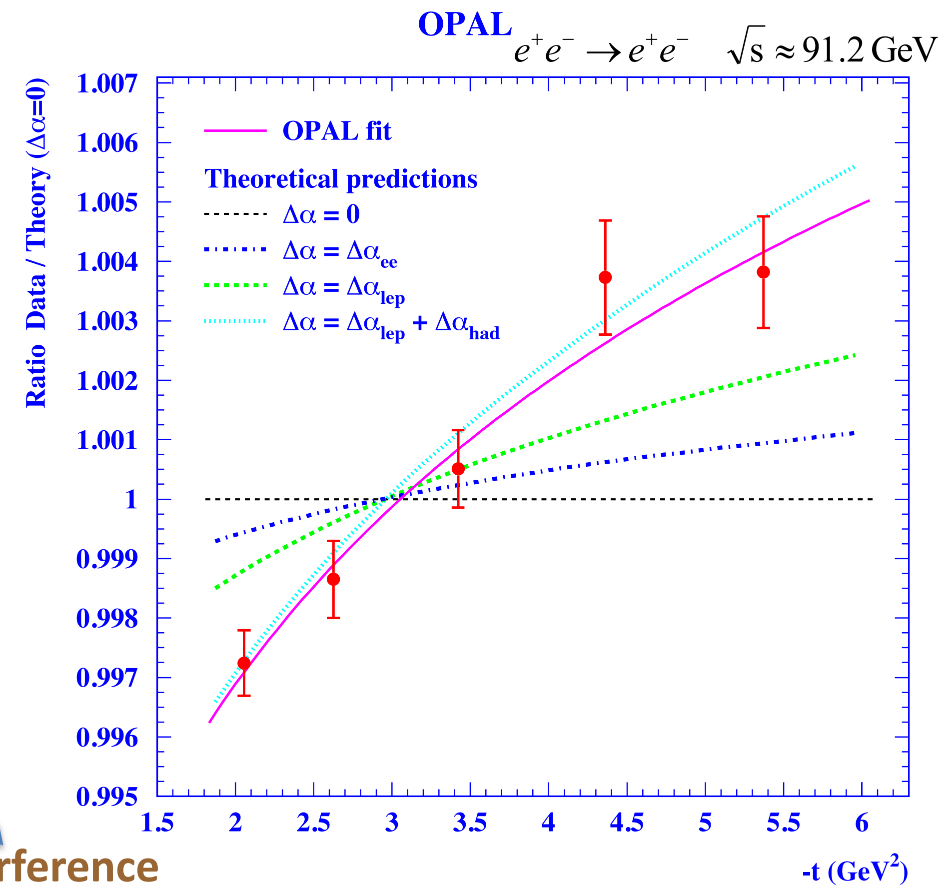
$$\left(\frac{1}{1-\Delta\alpha(t)}\right)^2$$

Effective coupling factorized

Photonic radiative corrections

s-channel γ exchange correction

Z interference correction



Other measurements in the space-like region by L3, VENUS