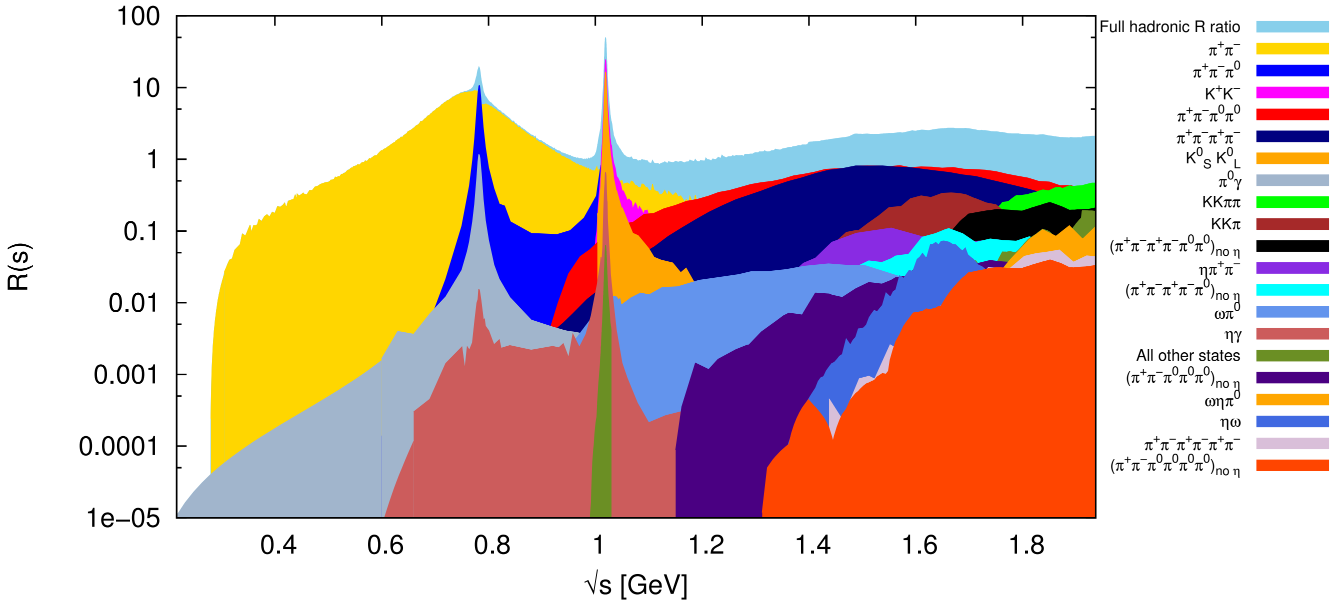
HVP: Dispersive Approach



Alex Keshavarzi

University of Manchester

Workshop on Muon Precision Physics

University of Liverpool

7th November 2022

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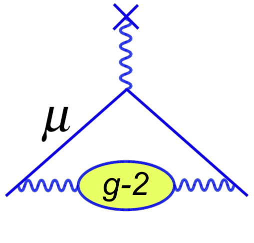

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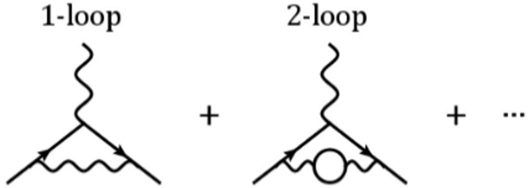
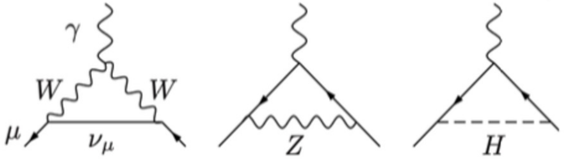
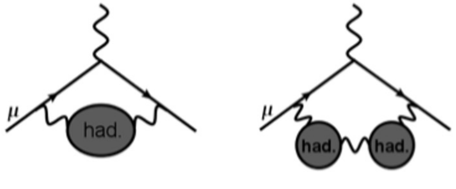
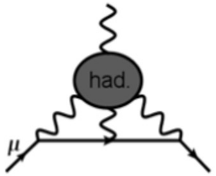
Dispersive HVP: the challenge

$$\Delta a_\mu = 279(76) \times 10^{-11} \rightarrow 2.39(0.65)\ \text{ppm}$$

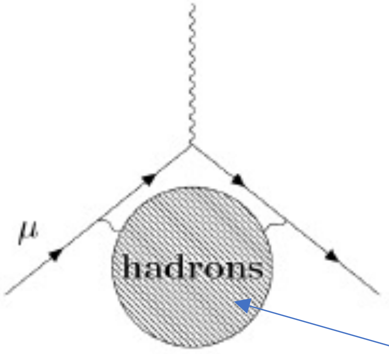
$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

- a_μ arises due to quantum corrections / higher order interactions / loop contributions
- All SM particles contribute → Calculate and sum all sectors of the SM.

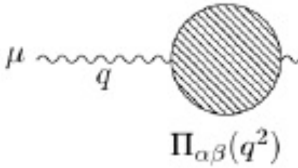
			a_μ^{SM} portion	δa_μ^{SM} portion
QED		Perturbative (Known to five-loop)	$\sim$ 99.99%	$\sim$0.001%
EW		Perturbative (Known to two-loop)	$\sim$ 1 ppm	$\sim$0.2%
HVP		Non-perturbative (Data-driven & lattice)	$\sim$ 59 ppm	$\sim$84%
HLbL		Non-perturbative (Data-driven & lattice)	$\sim$ 1 ppm	$\sim$16%

Dispersive HVP: the method

⇒ We want to calculate the leading order hadronic vacuum polarisation (HVP) contribution

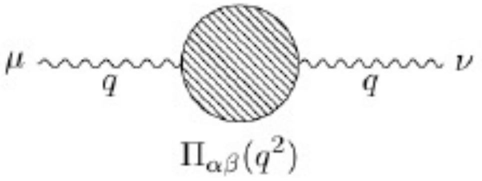


1) Feynman rules for HVP insertion to photon propagator:



$$= \frac{-ig^{\mu\alpha}}{(q^2 - i\varepsilon)}(-ie)i\Pi_{\alpha\beta}(q^2)(-ie)\frac{-ig^{\beta\nu}}{(q^2 - i\varepsilon)}$$

2) Employ analyticity:



$$= \frac{ie^2 g_{\mu\nu}}{(q^2 - i\varepsilon)^2}\frac{q^4}{\pi}\int_{s_{th}}^{\infty} \mathrm{d}s \frac{\mathrm{Im}\,\Pi(s)}{s(s - q^2 - i\varepsilon)}$$

3) Insert to vertex correction, solve for a_μ: $a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{\alpha}{\pi^2}\int_{s_{th}}^{\infty}\frac{\mathrm{d}s}{s}\mathrm{Im}\,\Pi_{\mathrm{had}}(s)K(s)$

4) Utilise optical theorem:

5) Arrive at equation for $a_\mu^{\mathrm{had,\,LO\,VP}}$:

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} \mathrm{d}s\, \sigma^0_{\mathrm{had},\gamma}(s)K(s)$$

$\sigma^0_{\mathrm{had},\gamma}$ = bare cross section, FSR included

Any and all permitted hadrons

Strongly weighted at low-energy (non-perturbative regime)

⇒ Similar dispersion integrals for NLO and NNLO HVP

The measured data

Dedicated measurements of $e^+e^- \to$ hadrons.

- $\lesssim$ 2 GeV = exclusive final states ($\pi^0\gamma, 2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 7\pi, K\bar{K}, K\bar{K}\pi, K\bar{K}2\pi, 2K\bar{K}, p\bar{p}, n\bar{n}$ ...).
- $\gtrsim$ 2 GeV = inclusive hadronic R-ratio (all hadrons).

Two methods from cross section measurement:

- Direct energy scan - fixed CM energy measurement of production cross section.
- Radiative return – measure differential cross section with tagged ISR photon to reconstruct production cross section.

Radiative Return

Babar ($E_{CM} = \Upsilon(4s)$)

- Comprehensive (almost all) exclusive final states measured below 2 GeV.
- High statistics, from-threshold measurements of $\pi^+\pi^-$.

KLOE ($E_{CM} = \phi$)

- 3 high-precision measurements of $\pi^+\pi^-$ on ρ-resonance, using different methods.
- Combination results in most precise measurement of $\pi^+\pi^-$.

BES-III ($E_{CM} =$ 2-5 GeV)

- High-precision measurement of $\pi^+\pi^-$ on ρ-resonance.
- Measurements of other modes, e.g. $\pi^+\pi^-\pi^0$, inclusive.

Others

- CLEO-c ($\pi^+\pi^-$).
- Belle-II (hopefully in the near future).

Direct scan

SND and CMD-3 (Novosibirsk)

- Both located at VEPP-2000 machine.
- Comprehensive (almost all) exclusive final states measured below 2 GeV.

KEDR (Novosibirsk)

- Inclusive measurement.

Plus, many older measurements from now inactive experiments…

We will hear more about these in the remaining talks today…

Radiative Corrections: MC Generators

We need high-precision MC generators for radiative corrections at the experiment level:

G. Venanzoni, Status of Radiative Corrections for e+e- data, Fifth Plenary Workshop of the Muon g-2 Theory Initiative

"Visible" cross section
$\sigma(e^+e^-(\gamma) \rightarrow X(\gamma))$

Here we correct for all detector effects

↓

Adjust for radiative corrections (ISR, FSR)
$\sigma(e^+e^- \rightarrow X)$

This one is used to get parameters of the resonances (mass, width,...)

Direct scan:

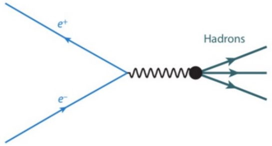
- For 2π, radiative corrections account for ISR and FSR effects.
- For non-2π:
  - Radiative correction accounts for ISR effects only.
  - Efficiency is calculated via Monte Carlo + corrections for imperfect detector.

Radiative return:

- Precise knowledge of ISR-process through radiator function is paramount.

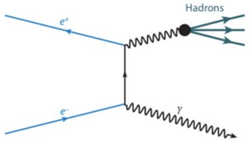
$$s \cdot \frac{d\sigma_{\pi\pi\gamma}}{ds_{\pi}} = \sigma_{\pi\pi}(s_{\pi}) \times H(s,s_{\pi})$$

MC generators for exclusive channels (exact NLO + Higher Order terms in some approx)



MC generator	Channel	Precision	Comment
MCGPJ (VEPP-2M, VEPP-2000)	$e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \pi^+\pi^-, \dots$	0.2%	photon jets along all particles (collinear Structure function) with exact NLO matrix elements
BabaYaga@NLO (KLOE, BaBar, BESIII)	$e^+e^- \rightarrow e^+e^-, \mu^+\mu^-, \gamma\gamma$	0.1%	QED Parton Shower approach with exact NLO matrix elements

MC generators for ISR (from approximate to exact NLO)



MC generator	Channel	Precision	Comment
EVA (KLOE)	$e^+e^- \rightarrow \pi^+\pi^-\gamma$	O(%)	Tagged photon ISR at LO + Structure Function FSR: point-like pions
AFKQED (BaBar)	$e^+e^- \rightarrow \pi^+\pi^-\gamma, \dots$	depends on the event selection (can be as good as Phokhara)	ISR at LO +Structure Function
PHOKHARA (KLOE, BaBar BESIII)	$e^+e^- \rightarrow \pi^+\pi^-\gamma, \mu^+\mu^-\gamma, 4\pi\gamma, \dots$	0.5%	ISR and FSR(sQED+Form Factor) at NLO

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$\sigma(e^+e^-(\gamma) \to X(\gamma))$

Here we correct for all detector effects

Adjust for ra
corrections (I
$\sigma(e^+e^- -$

Direct scan:
- For 2π, Radiative corrections acc ... and FSR effects.
- For non-2π:
... cts only.
... corrections for
... tor function is

Radiative corrections and MC generators for e+e- → hadrons, leptons should aim at 0.1% uncertainty. NNLO calculation needed!

In desperate need of people-power!

MC generator ... enerators for ISR (from approximate to exact NLO)

MC generator		recision	Comment
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Radiative Corrections: VP/FSR Corrections

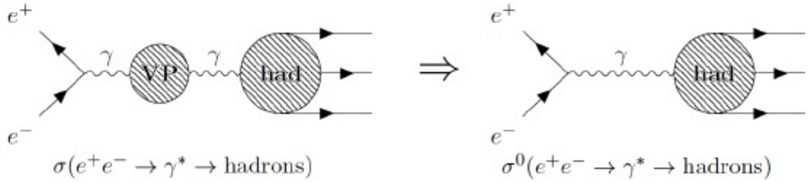
$\sigma^0_{\mathrm{had},\gamma}$ must be bare (undressed of VP effects) and inclusive of FSR effects. Must correct measured data not in this format:

⇒ Reconsider the optical theorem:

VP corrections

⇒ Photon VP corresponds to higher order contributions to $a_\mu^{\mathrm{had,\,VP}}$

→ Must subtract VP:



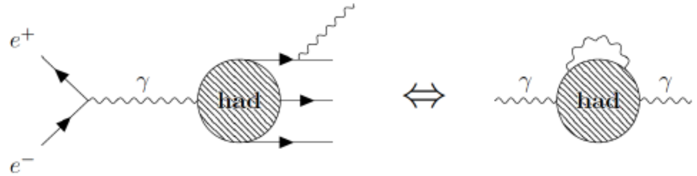
⇒ Fully updated, self-consistent VP routine: [vp_knt_v3_0], available for distribution

→ Cross sections undressed with full photon propagator (must include imaginary part), $\sigma^0_{\mathrm{had}}(s) = \sigma_{\mathrm{had}}(s)|1-\Pi(s)|^2$

⇒ If correcting data, apply corresponding radiative correction uncertainty

FSR corrections

⇒ Photon FSR formally higher order corrections to $a_\mu^{\mathrm{had,\,VP}}$



⇒ Cannot be unambiguously separated, not accounted for in HO contributions

→ Must be included as part of 1PI hadronic blobs

⇒ Experiment may cut/miss photon FSR → Must be added back

⇒ For $\pi^+\pi^-$, sQED approximation [Eur. Phys. J. C 24 (2002) 51, Eur. Phys. J. C 28 (2003) 261]

⇒ For higher multiplicity states, difficult to estimate correction ∴ Apply conservative uncertainty

No showstoppers here. Estimates between groups consistent and **very** conservative uncertainties applied.

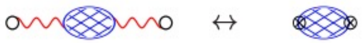
What about tau data?

From the 2020 Theory Initiative WP (*Phys.Rept.* 887 (2020) 1-166):

"at the required precision to match the e^+e^- data, the present understanding of the IB corrections to τ data is unfortunately not yet at a level allowing their use for the HVP dispersion integrals."

Recent claims that including $\rho - \gamma$ mixing can account for e.g. dispersive vs. lattice, Babar vs KLOE:

Commonly forgotten: mixing of ρ^0, ω, ϕ with the photon [$\rho^0 - \gamma$ mixing] i.e. effect concerning relation



$\langle A(x)\,A(0)\rangle$ ↔ $\langle j(x)\,j(0)\rangle$

photon propagator — current correlator

e^+e^- measurement ⇔ LQCD calculation

- how to disentangle QED from QCD in e^+e^--data ?
- $\rho^0 - \gamma$ absent in CC $\tau \to \nu_\tau \pi\pi$ data, but QED-QCD interference part incl. in $e^+e^- \to \pi^+\pi^-$ data,
- for getting had blob in e^+e^- the $\gamma - \rho^0$ mixing has to be removed!
- for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$ results in

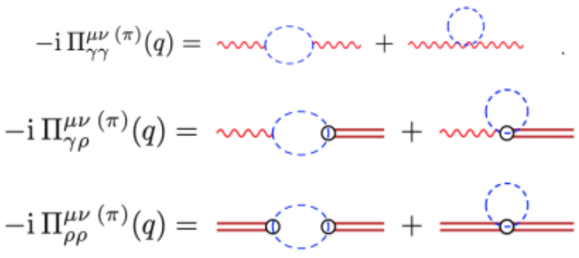
$$\delta a_\mu^{\rm had}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10}\,,$$

A critical assessment of Δα_QCD^had (mZ) and the prospects for improvements, F. Jegerlehner, ECFA Workshop on parametric uncertainties: α_em

Taking into account $\rho - \gamma$ interference resolves τ (charged channel) vs. e^+e^- (neutral channel) puzzle, F.J.& R. Szafron [JS11], M. Benayoun et al.. However, not accepted by WP as a possible effect, which is analogous to $Z - \gamma$ interference established at LEP in the 90's.

$\rho - \gamma$ interference
(absent in charged channel)
often mimicked by large shifts
in M_ρ and Γ_ρ
ρ^0 is mixing with γ:
propagators are obtained by
inverting the symmetric 2×2
self–energy matrix

$$\hat{D}^{-1} = \begin{pmatrix} q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) \end{pmatrix}.$$



Irreducible self-energy contribution at one-loop

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Recent claims that including $\rho-\gamma$ mixing can account for e.g. dispersive vs. lattice, Babar vs KLOE:

Commonly forgotten: mixing of ρ^0, ω, ϕ with the photon [$\rho^0-\gamma$ mixing] i.e. effect concerning relation

s for improvements, F.
ic uncertainties: α_em
tral channel)
P as a possible

1. **In a model-independent description of strong physics (QCD), the ρ is not a physical final state that you should account for in interaction with the photon. All production mechanisms effects are encapsulated in the final state.**
2. **There is a power counting issue. The $\rho-\gamma$ mixing diagram is part of the higher order HVP.**

- how to disenta ... QCD in e^+e^--data ?
- $\rho^0-\gamma$ absent in CC $\tau \to \nu_\tau \pi\pi$ data, but QED-QCD interference part incl. in $e^+e^- \to \pi^+\pi^-$ data,
- for getting had blob in e^+e^- the $\gamma-\rho^0$ mixing has to be removed!
- for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$ results in

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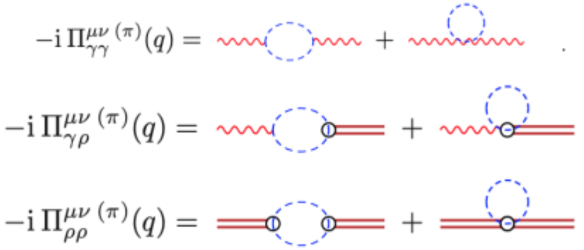
... often mimicked by large shifts in M_ρ and Γ_ρ
ρ^0 is mixing with γ: propagators are obtained by inverting the symmetric 2×2 self–energy matrix

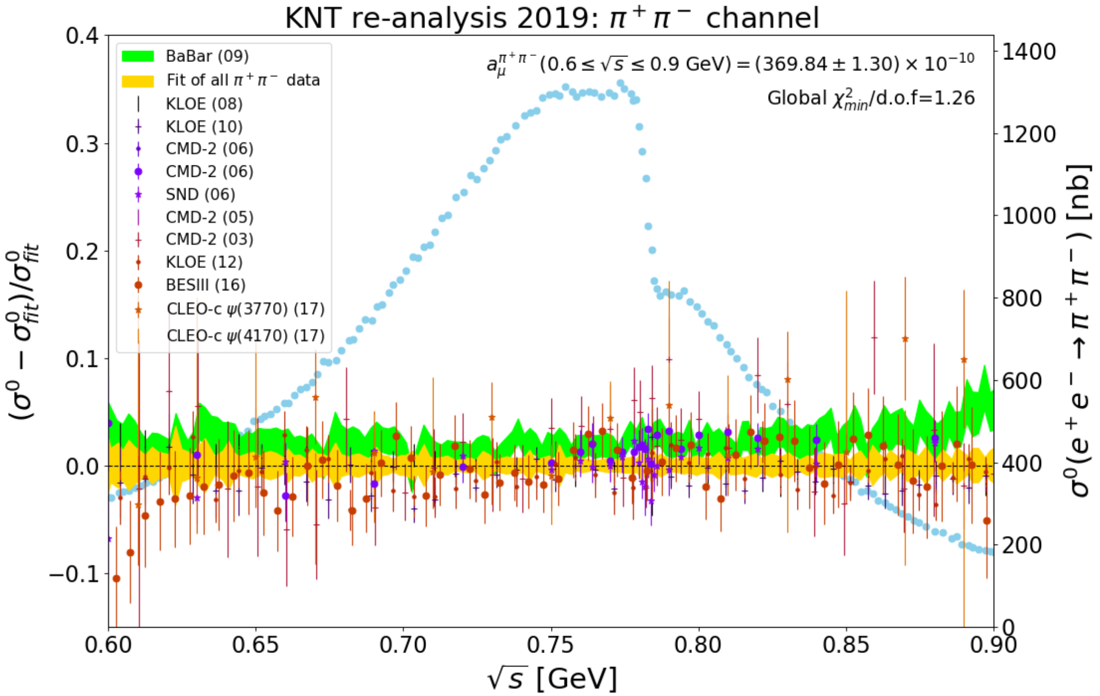
$$\hat{D}^{-1} = \begin{pmatrix} q^2+\Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2-M_\rho^2+\Pi_{\rho\rho}(q^2) \end{pmatrix}.$$

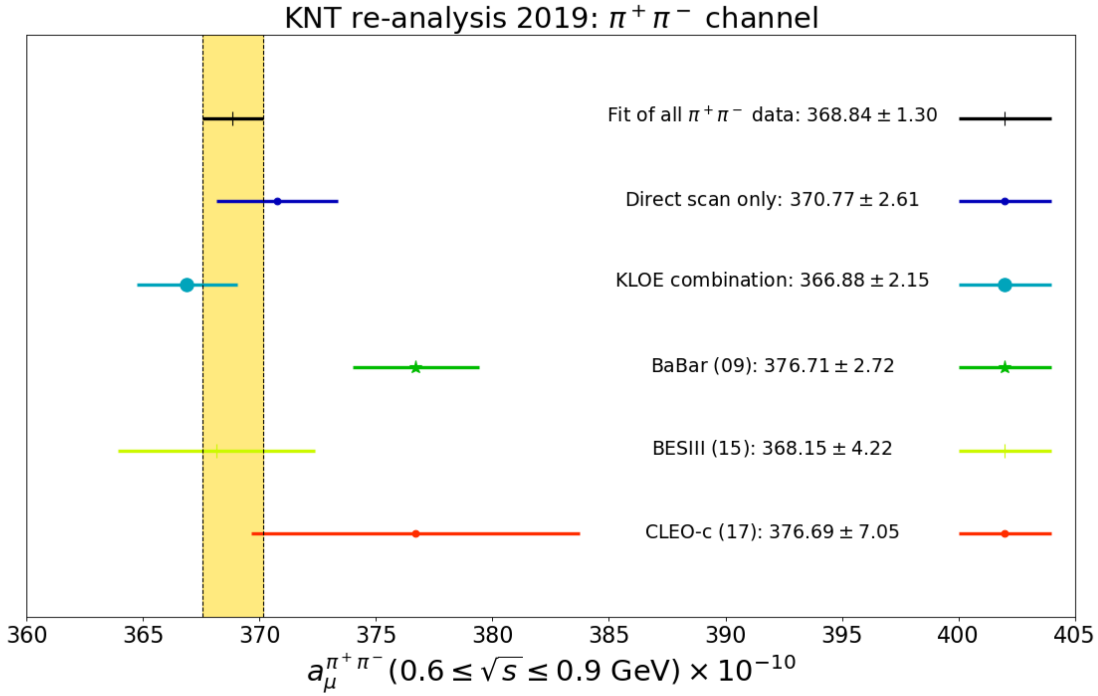


Irreducible self-energy contribution at one-loop

Data tensions, e.g. KLOE vs BaBar

Large difference between KLOE vs. BaBar is still evident, **but not at the level of the g-2 discrepancy!**





Compared to $a_\mu^{\pi^+\pi^-} = 503.5 \pm 1.9 \rightarrow a_\mu^{\pi^+\pi^-}$(BaBar data only) = 513.2 ± 3.8

Simple weighted average of all data → $a_\mu^{\pi^+\pi^-}$(weighted average) = 509.2 ± 2.9

(i.e. – no correlations in determination of mean value)

BaBar data dominate when no correlations are accounted for in the mean value.

- ➢ Highlights the importance of incorporating available correlated uncertainties in fit.

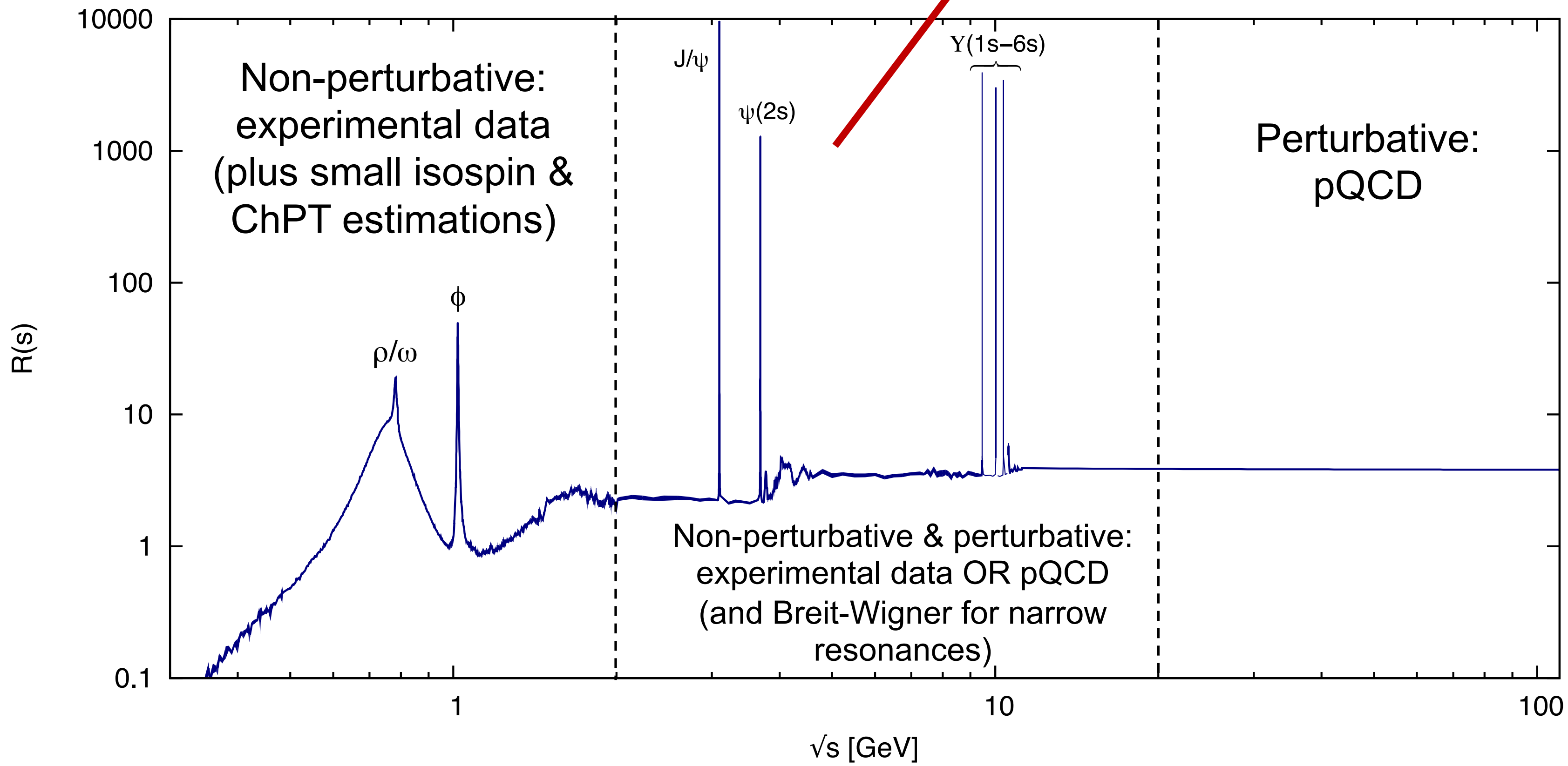
- Data tensions also present in other channels.
- Accounted for with error inflation and additional uncertainties.

Dispersive HVP: the real challenge

- Target: ~ 0.2% total error.
- Current dispersive uncertainty: ~ 0.5%.
- Below ~ 2 GeV:
  - Radiative corrections.
  - Combine data for > 50 exclusive channels.
  - Use isospin / ChPT relations for missing channels (tiny, < 0.05%).
  - Sum all channels for total cross section.
- Above ~ 2 GeV:
  - Combine inclusive data OR pQCD (away from flavour thresholds).
  - Add narrow resonances.
- Challenges:
  - How to combine data/errors/correlations from different experiments and measurements.
  - Accounting for tensions & sources of systematic error.

$$a_\mu^{\mathrm{had,\,LO\,VP}} = \frac{1}{4\pi^3}\int_{s_{th}}^{\infty} \mathrm{d}s\, \sigma^0_{\mathrm{had},\gamma}(s) K(s)$$

$$R(s) = \sigma^0_{had}(s)/\left(\frac{4\pi\alpha^2}{3s}\right)$$



Phys.Rev.D 97 (2018) 114025, Phys.Rev.D 101 (2020) 014029.

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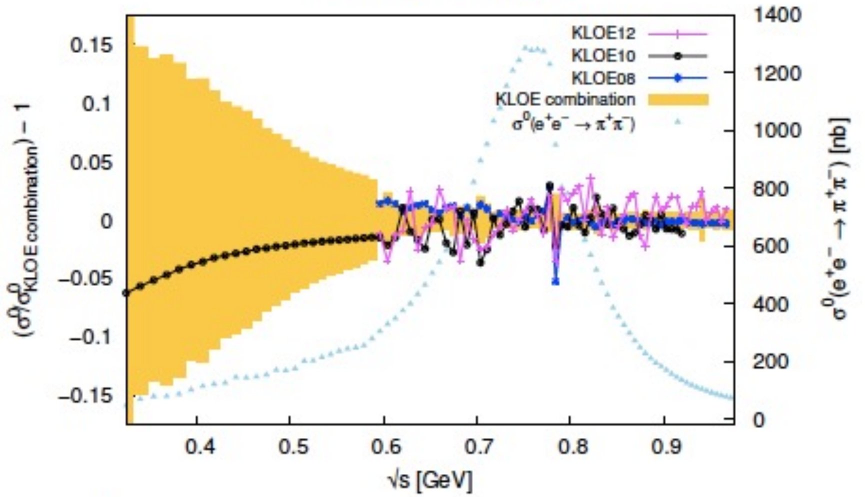
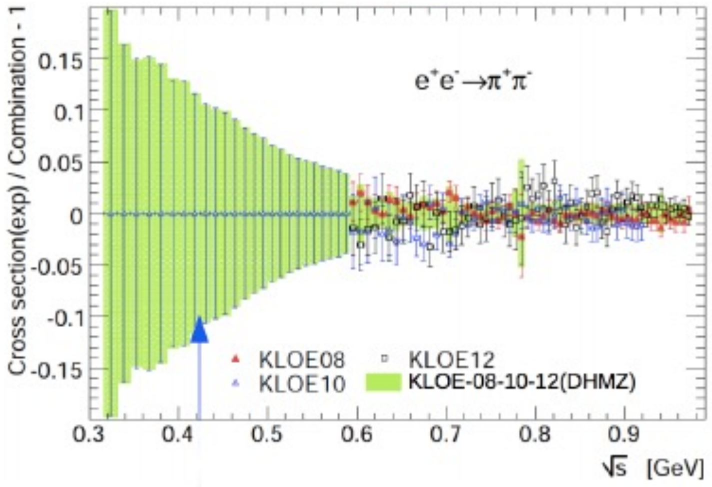
Analysis approaches: DHMZ & KNT

Analysis step	KNT (*Phys.Rev.D* 97 (2018) 114025, *Phys.Rev.D* 101 (2020) 014029)	DHMZ (Eur. Phys. J. C80, 241 (2020), [Erratum: Eur. Phys. J. C80, 410 (2020)])
Blinding	Included for upcoming update	None
VP Correction	Self-consistent VP routine + conservative uncertainty.	Self-consistent VP routine + some uncertainty (?).
FSR corrections	Scalar QED for two body + conservative uncertainty.	Scalar QED for two body + some uncertainty (?).
Re-binning	Re-bin data into "clusters". Scans over cluster configurations for optimisation.	Quadratic splines of all data sets quadratically interpolated on fixed binning.
Additional constraints	None.	Analyticity constraints for 2π channel.
Fitting	χ^2 minimisation with correlated uncertainties incorporated **globally.**	χ^2 minimisation with correlated uncertainties incorporated **locally**.
Error inflation	Local χ^2 error inflation.	Local χ^2 error inflation.
Integration	Trapezoidal for continuum, quintic for resonances.	Quadratic interpolation.

$a_\mu^{\pi^+\pi^-}(\sqrt{s} < 2\ \text{GeV}) = 503.74 \pm 1.96$





$a_\mu^{\pi^+\pi^-}(\sqrt{s} < 2\ \text{GeV}) = 507.14 \pm 2.58$

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Other analyses and choices

Phys.Rept. 887 (2020) 1-166.

Energy range	ACD18	CHS18	DHMZ19	DHMZ19′	KNT19
≤ 0.6 GeV		110.1(9)	110.4(4)(5)	110.3(4)	108.7(9)
≤ 0.7 GeV		214.8(1.7)	214.7(0.8)(1.1)	214.8(8)	213.1(1.2)
≤ 0.8 GeV		413.2(2.3)	414.4(1.5)(2.3)	414.2(1.5)	412.0(1.7)
≤ 0.9 GeV		479.8(2.6)	481.9(1.8)(2.9)	481.4(1.8)	478.5(1.8)
≤ 1.0 GeV		495.0(2.6)	497.4(1.8)(3.1)	496.8(1.9)	493.8(1.9)
[0.6, 0.7] GeV		104.7(7)	104.2(5)(5)	104.5(5)	104.4(5)
[0.7, 0.8] GeV		198.3(9)	199.8(0.9)(1.2)	199.3(9)	198.9(7)
[0.8, 0.9] GeV		66.6(4)	67.5(4)(6)	67.2(4)	66.6(3)
[0.9, 1.0] GeV		15.3(1)	15.5(1)(2)	15.5(1)	15.3(1)
≤ 0.63 GeV	132.9(8)	132.8(1.1)	132.9(5)(6)	132.9(5)	131.2(1.0)
[0.6, 0.9] GeV		369.6(1.7)	371.5(1.5)(2.3)	371.0(1.6)	369.8(1.3)
[$\sqrt{0.1}$, $\sqrt{0.95}$] GeV		490.7(2.6)	493.1(1.8)(3.1)	492.5(1.9)	489.5(1.9)

Analyticity constraints

JHEP 02, 006 (2019). JHEP 08, 137 (2019). Eur. Phys. J. C80, 241 (2020). Eur. Phys. J. C80, 410 (2020)].

- Constraints to hadronic cross section applied from analyticity, unitarity, and crossing symmetry.
- These allow derivations of global fit functions based on fundamental properties of QCD.
- Can lead to reduction in uncertainties.
- Successfully applied for $2\pi, 3\pi, \pi^0\gamma$ channels.

Fred Jegerlehner's combination

- Data-sets from the same experiment are combined in local regions of $\sqrt{s}$ using a global χ^2 minimisation.
- Overlapping regions of combined data are then averaged.
- Resonances are parameterised using models (e.g. G-S, BW), with masses are fixed to PDG values.
- τ data are/aren't included. Isospin corrections are made for e.g. $\rho - \gamma$ mixing.

F. Jegerlehner, EPJ Web Conf. 199, 01010 (2019), arXiv:1809.07413 [hep-ph].

Broken Hidden Local Symmetry (Benyanoun, Jegerlehner)

- Effective Lagrangian based on vector meson dominance and resonance ChPT.
- BHLS model parameters are extracted from experimental data.
- Can lead to drastically reduced uncertainties, but some data must be discarded.

M. Benayoun, L. Delbuono, and F. Jegerlehner, Eur. Phys. J. C80, 81 (2020), [Erratum: Eur. Phys. J. C80, 244 (2020)], arXiv:1903.11034 [hep-ph].

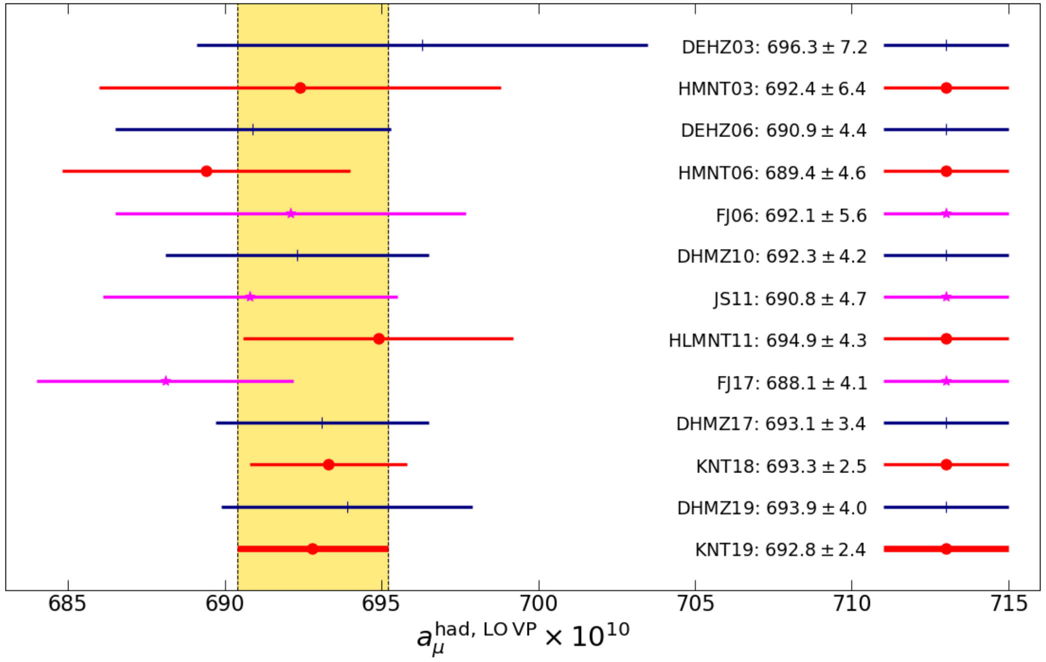
	BDJ19	DHMZ19	FJ17	KNT19
$a_\mu^{\mathrm{HVP,\,LO}} \times 10^{10}$	687.1(3.0)	694.0(4.0)	688.1(4.1)	692.8(2.4)

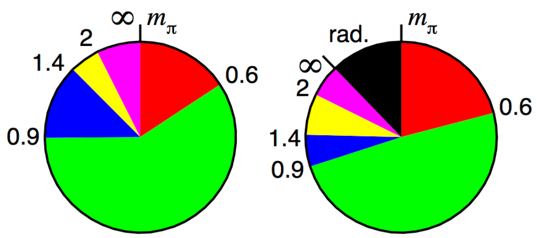
Comparisons and the 2021 WP result

KNT19, *Phys.Rev.D* 97 (2018) 114025, *Phys.Rev.D* 101 (2020) 014029.

$$a_\mu^{\text{had, LOVP}} = 693.84 \pm 1.19_{stat} \pm 1.96_{sys} \pm 0.22_{vp} \pm 0.71_{fsr}$$

$$= 692.78 \pm 2.42_{tot}$$





- ➢ Clear $\pi^+\pi^-$ dominance

- ➢ Precision better than 0.4% (uncertainties include all available correlations and χ^2 inflation)

Phys.Rept. 887 (2020) 1-166.

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	694.0(1.0)(3.5)(1.6)(0.1)$_\psi$(0.7)$_{\text{DV+QCD}}$	692.8(2.4)	1.2

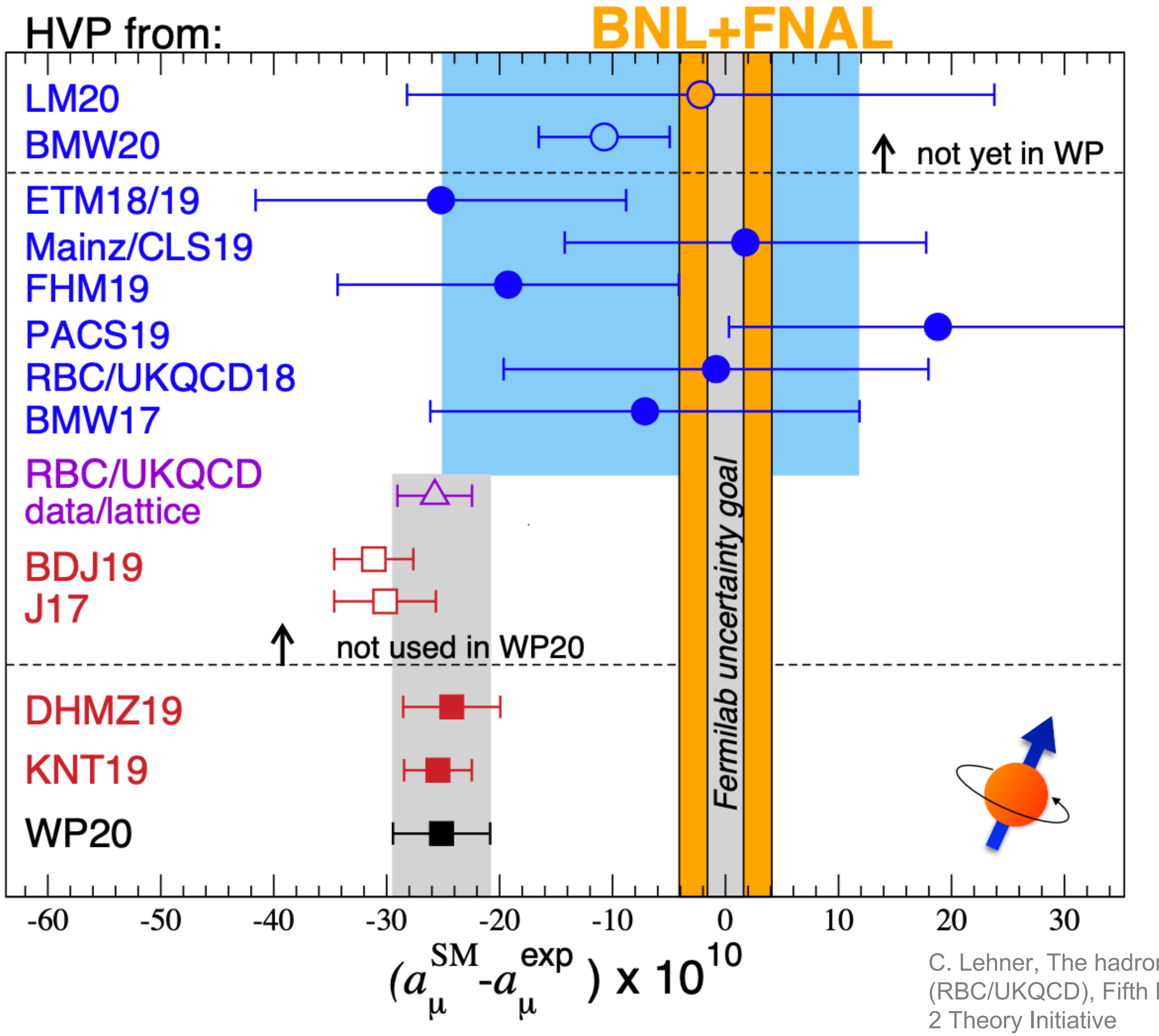
+ evaluations using unitarity & analyticity constraints for $\pi\pi$ and $\pi\pi\pi$ channels [CHS 2018, HHKS 2019]

Conservative merging to obtain a realistic assessment of the underlying uncertainties:

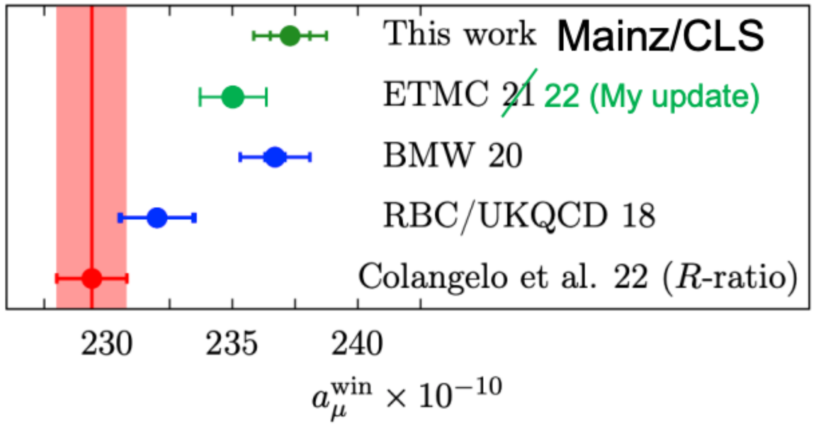
- Account for differences in results from the same experimental inputs.
- Include correlations between systematic errors

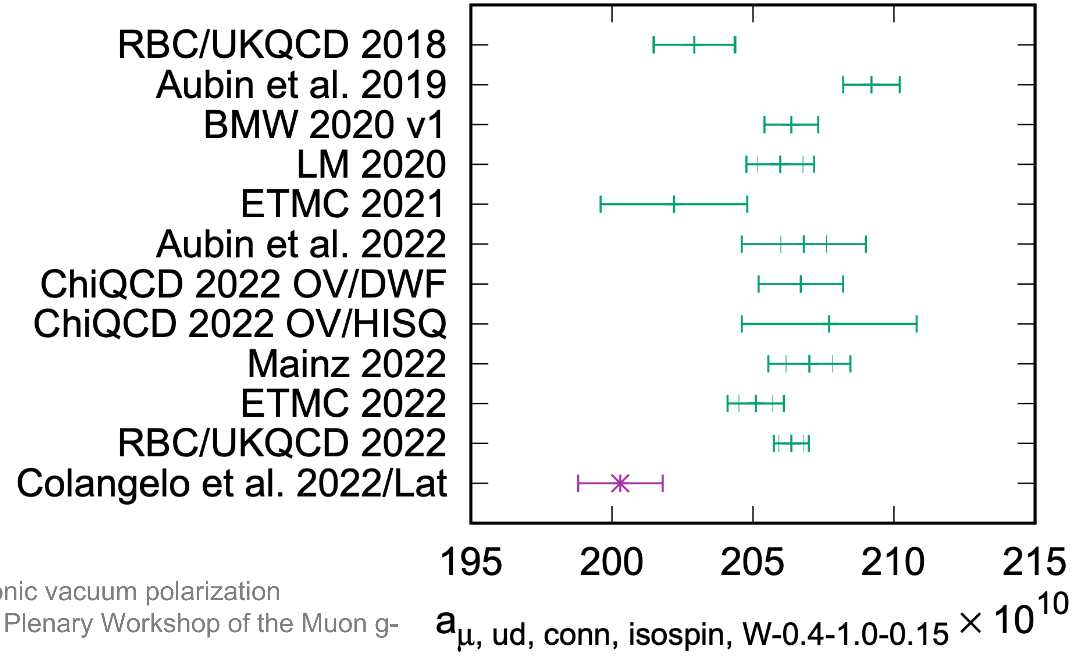
$$\Rightarrow a_\mu^{\text{HVP,LO}} = 693.1\,(4.0) \times 10^{-10}$$

The importance of the HVP for Δa_μ



https://arxiv.org/abs/2206.06582 Jun 14th 2022





C. Lehner, The hadronic vacuum polarization (RBC/UKQCD), Fifth Plenary Workshop of the Muon g-2 Theory Initiative

Connection with $\Delta\alpha_{\text{had}}$

- $\Delta\alpha_{\text{had}}$ limits precision of EW precision fits and so the effectiveness of high-precision EW measurements.
- Can draw a direct parallel with evaluation of the Muon g-2 and probe the muon g-2 discrepancy.
- Is a test of low-energy hadronic theory, e.g. Lattice QCD vs dispersive e^+e^- data.

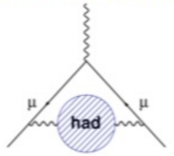
Uncertainty from e^+e^- data $\sim 0.5\%$

Parameter	Input value	Fit result	Result w/o input value
M_W (GeV)	80.379(12)	80.359(3)	80.357(4)(5)
M_H (GeV)	125.10(14)	125.10(14)	94^{+20+6}_{-18-6}
$\Delta\alpha^{(5)}_{\text{had}}(M_Z^2) \times 10^4$	276.1(1.1)	275.8(1.1)	272.2(3.9)(1.2)
m_t (GeV)	172.9(4)	173.0(4)	...
$\alpha_s(M_Z^2)$	0.1179(10)	0.1180(7)	...
M_Z (GeV)	91.1876(21)	91.1883(20)	...
Γ_Z (GeV)	2.4952(23)	2.4940(4)	...
Γ_W (GeV)	2.085(42)	2.0903(4)	...
σ^0_{had} (nb)	41.541(37)	41.490(4)	...
R^0_l	20.767(25)	20.732(4)	...
R^0_c	0.1721(30)	0.17222(8)	...
R^0_b	0.21629(66)	0.21581(8)	...
$\bar{m}_c$ (GeV)	1.27(2)	1.27(2)	...
$\bar{m}_b$ (GeV)	$4.18^{+0.03}_{-0.02}$	$4.18^{+0.03}_{-0.02}$	...
$A^{0,l}_{\text{FB}}$	0.0171(10)	0.01622(7)	...
$A^{0,c}_{\text{FB}}$	0.0707(35)	0.0737(2)	...
$A^{0,b}_{\text{FB}}$	0.0992(16)	0.1031(2)	...
A_ℓ	0.1499(18)	0.1471(3)	...
A_c	0.670(27)	0.6679(2)	...
A_b	0.923(20)	0.93462(7)	...
$\sin^2\theta^{\text{lep}}_{\text{eff}}(Q_{\text{FB}})$	0.2324(12)	0.23152(4)	0.23152(4)(4)
$\sin^2\theta^{\text{lep}}_{\text{eff}}$(Had Coll)	0.23140(23)	0.23152(4)	0.23152(4)(4)

Keshavarzi, Marciano, Passera and Sirlin, *Phys.Rev.D* 102 (2020) 3, 033002

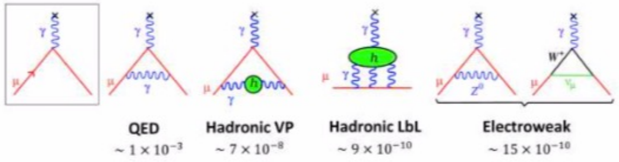
Experimentally measured hadronic cross section: σ_{had}

Muon g-2: hadronic vacuum polarisation contribution



$$a_\mu^{\text{had, VP}} = \frac{1}{4\pi^3}\int_{m_\pi}^{\infty} ds\ \sigma_{\text{had}}(s)\,K(s)$$

… sum with other SM contributions…



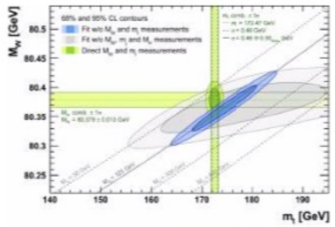
→ Determines a_μ^{SM} and $\Delta a_\mu = 3.7\sigma$

Increase cross section so that $\Delta a_\mu = 0$?
→ Solves muon g-2 discrepancy

Running QED coupling: hadronic contribution to running

$$\Delta\alpha^{(5)}_{\text{had}}(q^2) = \frac{q^2}{4\pi\alpha^2}\int_{m_\pi}^{\infty} ds\ \sigma_{\text{had}}(s)\frac{q^2}{(q^2-s)}$$

… evaluate at $q^2 = M_Z^2$ and input into **global EW fit**…



→ Predicts $M_W, M_H, \sin^2\theta^{lep}_{eff}$ and more…

Increase cross section so that $\Delta a_\mu = 0$?
→ What happens to precision EW parameters?

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The muon g-2 and $\Delta\alpha$ connection

Keshavarzi, Marciano, Passera and Sirlin, *Phys.Rev.D* 102 (2020) 3, 033002

- Shift KNT hadronic cross section in fully energy-dependent (point-like and binned) analysis to account for Δa_μ.
- Input new values of $\Delta\alpha$ into Gfitter to predict EW observables.
- Analysis greatly constrained from more precise EW observables measurements and more comprehensive hadronic cross section.
  - **Can Δa_μ be due to hypothetical mistakes in the hadronic $\sigma(s)$?**
  - **An upward shift of $\sigma(s)$ also induces an increase of $\Delta\alpha_{had}^{(5)}(M_Z)$.**
  - **Consider:**

$a_\mu^{HLO} \rightarrow$
$$a = \int_{4m_\pi^2}^{s_u} ds\, f(s)\,\sigma(s), \qquad f(s) = \frac{K(s)}{4\pi^3},\ s_u < M_Z^2,$$

$\Delta\alpha_{had}^{(5)} \rightarrow$
$$b = \int_{4m_\pi^2}^{s_u} ds\, g(s)\,\sigma(s), \qquad g(s) = \frac{M_Z^2}{(M_Z^2 - s)(4\alpha\pi^2)},$$

and the increase

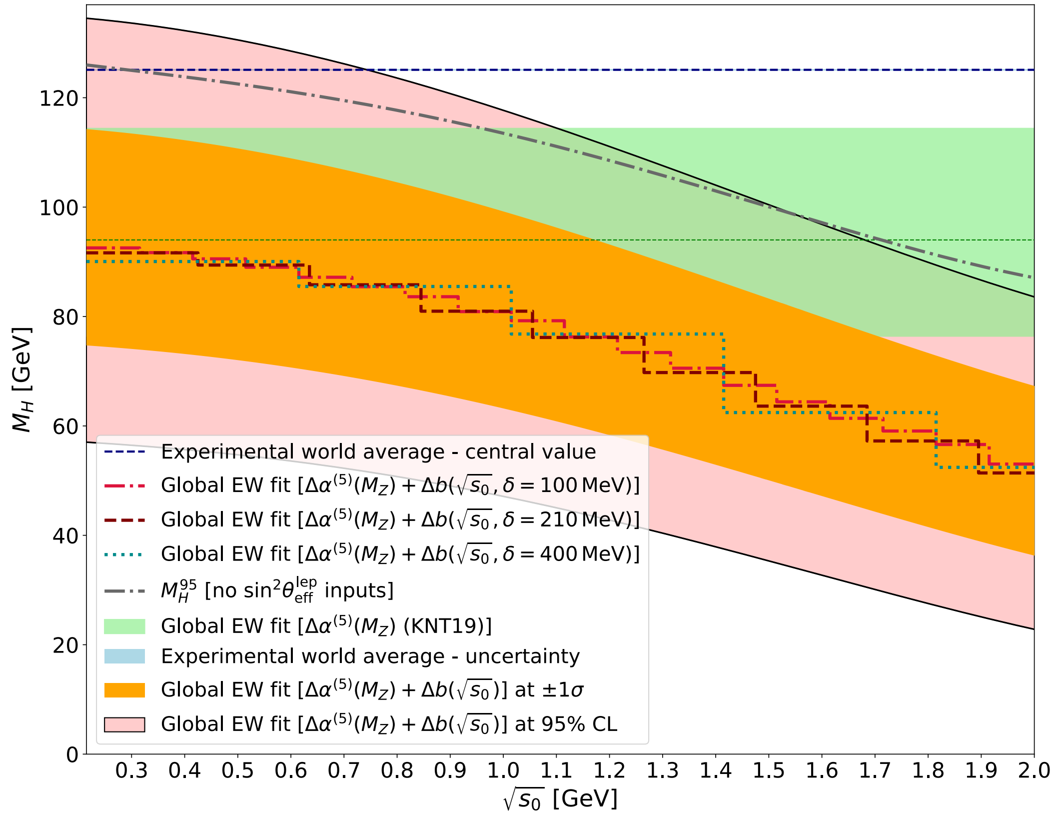
$$\Delta\sigma(s) = \epsilon\sigma(s)$$

Note the very different energy-dependent weighting of the integrands…

ϵ>0, in the range:

$$\sqrt{s} \in [\sqrt{s}_0 - \delta/2, \sqrt{s}_0 + \delta/2]$$

Use Gfitter and precise and up-to-date compilation of total hadronic cross section from KNT, Keshavarzi, Nomura and Teubner, Phys.Rev.D 101 (2020) 014029.

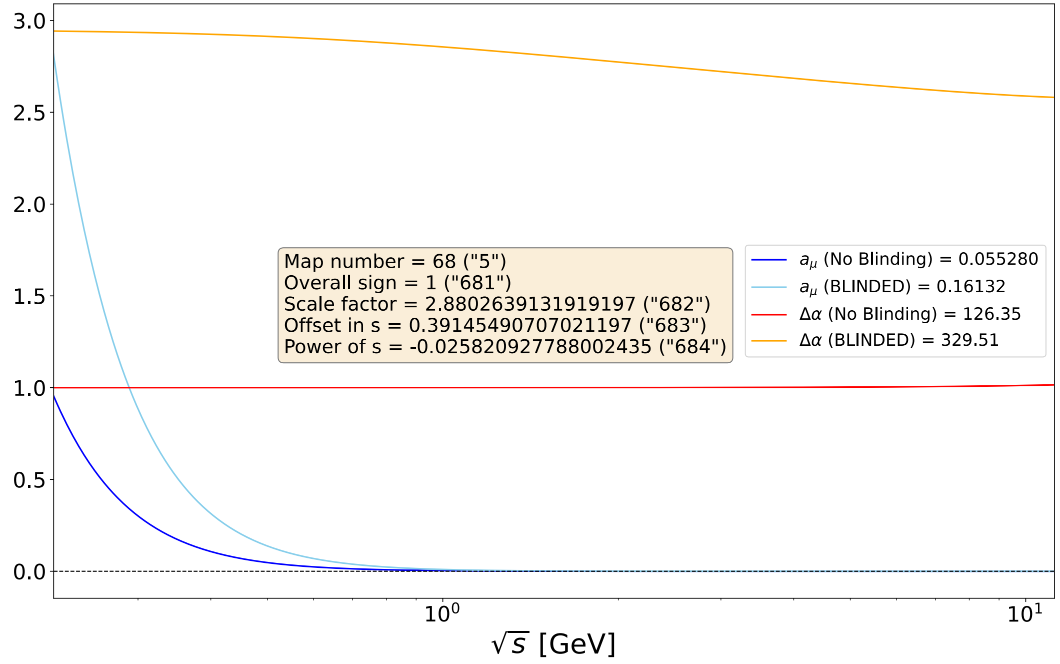


Shifting $\Delta\sigma(s)$ to fix Δa_μ is possible, but:

- Excluded above ~ 1 GeV.
- Increases to cross section needed are orders of magnitude larger than experimental uncertainties.

Plans and prospect for improvements

- New data:
  - New two-pion measurements from CMD-3 imminent.
  - Also, high-stats two-pion data from BaBar/KLOE, and hopefully from Belle-2.
  - Measurements expected for other channels, issues to be resolved in three-pions.
- Analysis choices:
  - Blinding. This is now implemented for KNT.
  - Updates to combination, fitting etc.
  - Modern hadronic cross section database.
  - Updated software (e.g. FORTRAN --> python).
- $\Delta\alpha_{\text{had}}$:
  - $\Delta\alpha_{\text{had}}$ improvements also possible via e.g. data smoothing.
  - Full delta alpha analysis long-planned from KNT. Full update to software package intended.
- Comparisons with lattice:
  - Up-to-date values for Euclidean windows.



Conclusions

- SM prediction is now entirely limited by HVP.
- This is worsened by the current dispersive vs lattice discrepancies.
- Strong and robust programme of consistent hadronic cross sections from decades of measurements from different experiments → more to come.
- Work needed to improve MC generators for experimental radiative corrections.
- Data tensions exist but covered by additional uncertainties.
- Several options for analysis choices by different groups. These lead to some different results.
- But, various HVP dispersive evaluations have been consistent for decades. No sign of this changing.
- "Allowed" changes to the hadronic cross section to account for the known discrepancies are orders of magnitude larger than experimental uncertainties.
- Plans to improve dispersive HVP further are underway. Aiming for 0.2% uncertainty.

In general, zero indication that there is anything missing, incorrect or misunderstood in dispersive HVP.