HVP from Lattice QCD

Christoph Lehner
(Regensburg)

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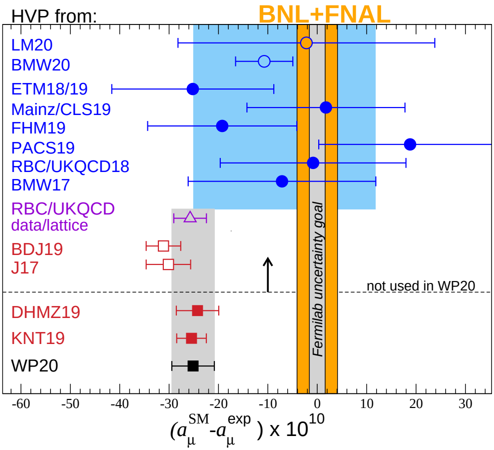
Status and impact of hadronic vacuum polarization contribution



Ab-initio lattice QCD(+QED) calculations are maturing

Difficult problem: scales from $2m_\pi$ to several GeV enter; cross-checks needed at high precision

Hybrid window method restricts scales that enter from lattice/dispersive data

Dispersive, $e^+e^- \to$ hadrons (20+ years of experiments)

Now first published lattice result with sub-percent precision available (BMW20), cross-checks are crucial to establish or refute high-precision lattice methodology

Summary of HVP status:

- Decades of e^+e^- dispersive results suggest a strong tension (4.2σ)
- A first sub-percent precision lattice result (BMW20) suggests only minimal tension (1.5σ)

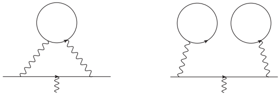
Two main questions addressed in this talk:

- Consistency of BMW20 lattice result with other lattice results
- Consistency of lattice results with R-ratio

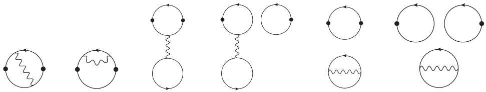
Consistency of BMW20 lattice result with other lattice results

Diagrams

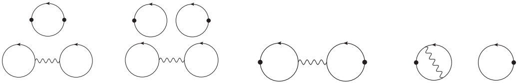
Isospin limit



QED corrections

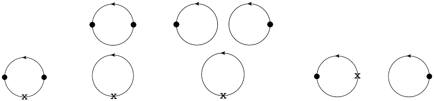


(a) V (b) S (c) T (d) T_d (e) D1 (f) $D1_d$



(g) D2 (h) $D2_d$ (i) F (j) D3

Strong isospin breaking



(a) M (b) R (c) R_d (d) O

Overview of individual contributions

Diagrams – Isospin limit

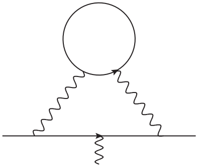
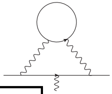
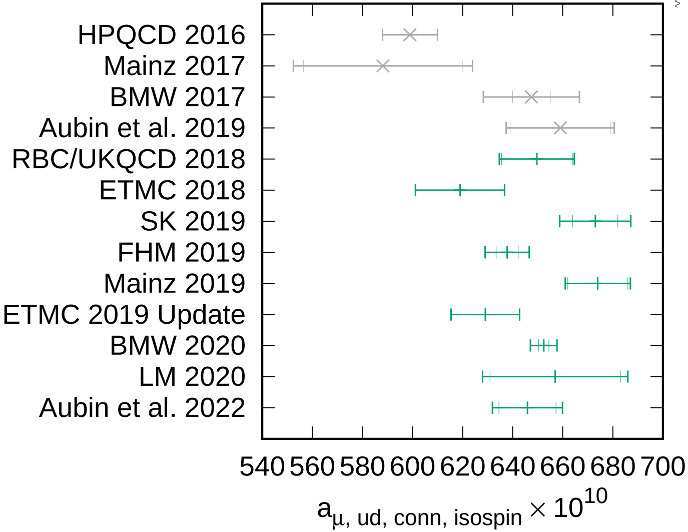


FIG. 1. Quark-connected (left) and quark-disconnected (right) diagram for the calculation of $a_\mu^{\mathrm{HVP\ LO}}$. We do not draw gluons but consider each diagram to represent all orders in QCD.

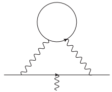
Up, down; isospin symmetric limit; $m_\pi = m_\pi^0$

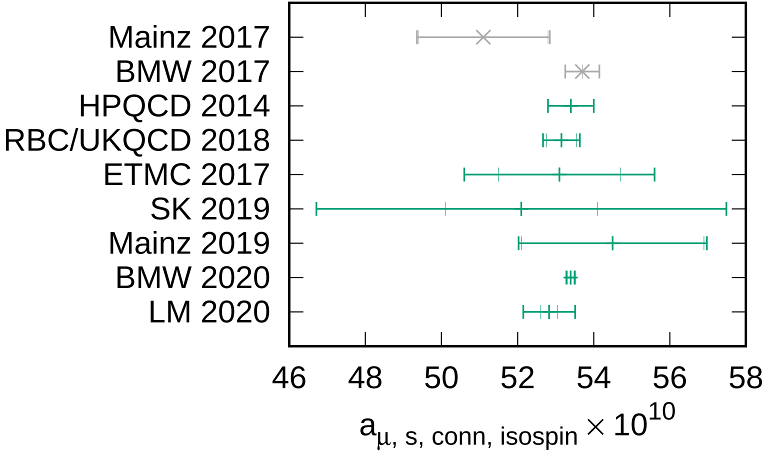




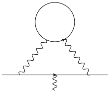
Some tensions to be understood

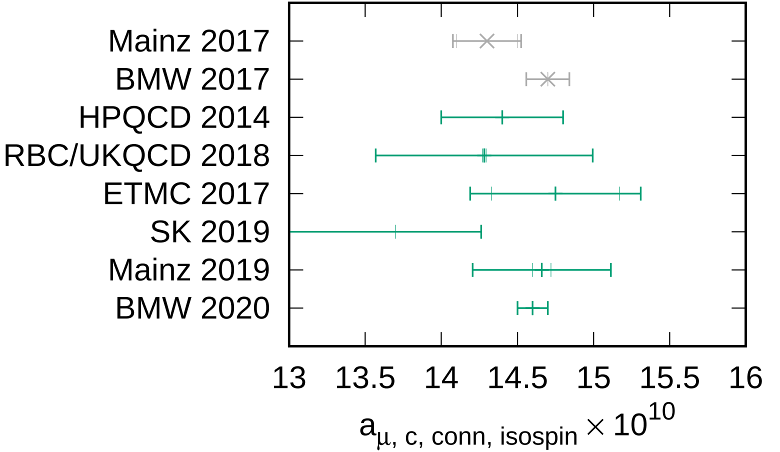
Strange

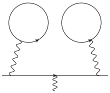


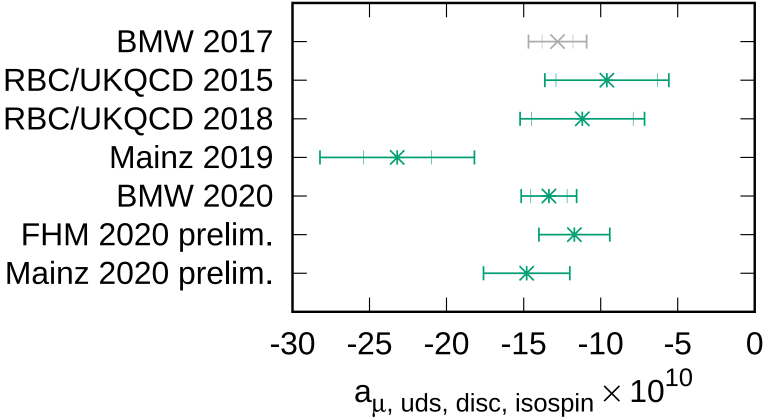

Mainz 2017
BMW 2017
HPQCD 2014
RBC/UKQCD 2018
ETMC 2017
SK 2019
Mainz 2019
BMW 2020
LM 2020
46
48
50
52
54
56
58
$a_{\mu,\, s,\, conn,\, isospin} \times 10^{10}$

Charm

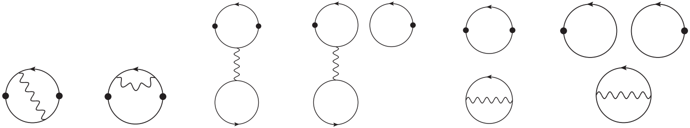



Mainz 2017
BMW 2017
HPQCD 2014
RBC/UKQCD 2018
ETMC 2017
SK 2019
Mainz 2019
BMW 2020
13
13.5
14
14.5
15
15.5
16
$a_{\mu,\ c,\ conn,\ isospin} \times 10^{10}$

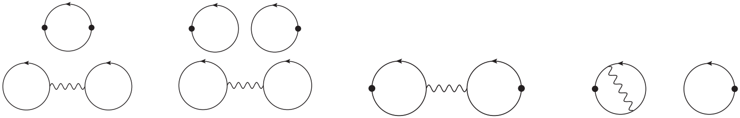



BMW 2017
RBC/UKQCD 2015
RBC/UKQCD 2018
Mainz 2019
BMW 2020
FHM 2020 prelim.
Mainz 2020 prelim.
-30 -25 -20 -15 -10 -5 0
$a_{\mu,\text{ uds, disc, isospin}} \times 10^{10}$

Diagrams – QED corrections

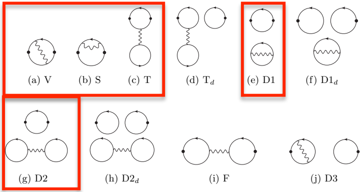


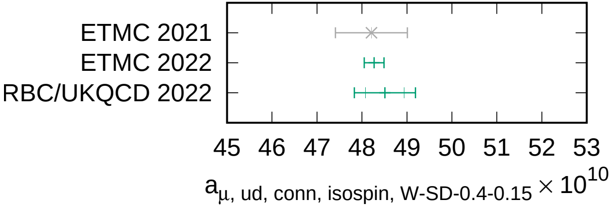
(a) V (b) S (c) T (d) T_d (e) D1 (f) $\mathrm{D1}_d$

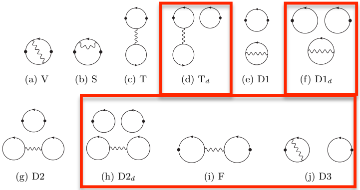


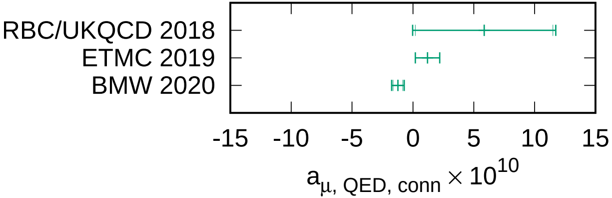
(g) D2 (h) $\mathrm{D2}_d$ (i) F (j) D3

For diagram F we enforce exchange of gluons between the quark loops as otherwise a cut through a single photon line would be possible. This single-photon contribution is counted as part of the HVP NLO and not included for the HVP LO.


(a) V
(b) S
(c) T
(d) T_d
(e) D1
(f) $D1_d$
(g) D2
(h) $D2_d$
(i) F
(j) D3

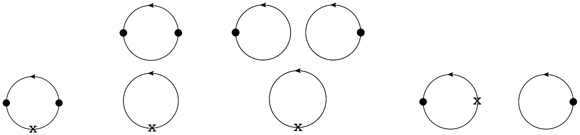

ETMC 2021
ETMC 2022
RBC/UKQCD 2022
45 46 47 48 49 50 51 52 53
$a_{\mu,\text{ ud, conn, isospin, W-SD-0.4-0.15}} \times 10^{10}$





Attention needed

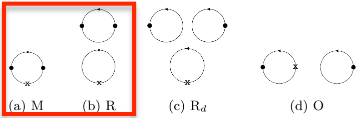
Diagrams – Strong isospin breaking



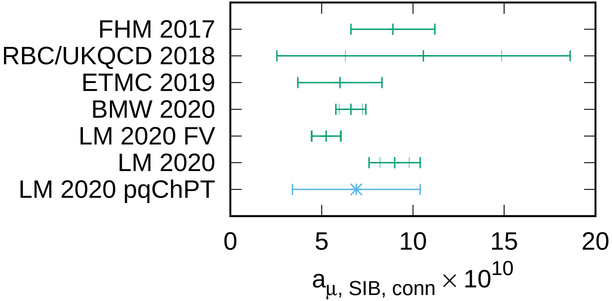
(a) M (b) R (c) R_d (d) O

For the HVP R is negligible since $\Delta m_u \approx -\Delta m_d$ and O is SU(3) and $1/N_c$ suppressed.

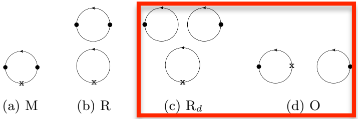
Lehner, Meyer 2020: NLO PQChPT: FV effects in connected and disconnected cancel but are each significant $O(4 \times 10^{-10})$; PQChPT expects cancellation between connected and disconnected contribution $a_\mu^{\text{SIB, conn.}} = -a_\mu^{\text{SIB, disc.}} = 6.9 \times 10^{-10}$

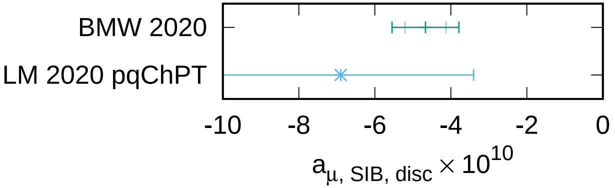


(a) M
(b) R
(c) R$_d$
(d) O



FHM 2017
RBC/UKQCD 2018
ETMC 2019
BMW 2020
LM 2020 FV
LM 2020
LM 2020 pqChPT
0
5
10
15
20
$a_{\mu,\text{ SIB, conn}} \times 10^{10}$


(a) M
(b) R
(c) R_d
(d) O


BMW 2020
LM 2020 pqChPT
-10
-8
-6
-4
-2
0
$a_{\mu,\text{ SIB, disc}} \times 10^{10}$

Attention on light-quark isospin-symmetric contribution and QED disconnected contribution

Lattice QCD – Time-Moment Representation

Starting from the vector current $J_\mu(x) = i\sum_f Q_f \overline{\Psi}_f(x)\gamma_\mu\Psi_f(x)$ we may write

$$a_\mu^{\mathrm{HVP\ LO}} = \sum_{t=0}^{\infty} w_t C(t)$$

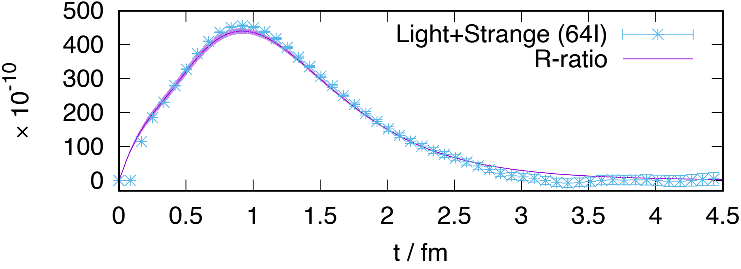
with

$$C(t) = \frac{1}{3}\sum_{\vec{x}}\sum_{j=0,1,2}\langle J_j(\vec{x},t)J_j(0)\rangle$$

and w_t capturing the photon and muon part of the HVP diagrams (Bernecker-Meyer 2011).

The correlator $C(t)$ is computed in lattice QCD+QED at physical pion mass with non-degenerate up and down quark masses including up, down, strange, and charm quark contributions. The missing bottom quark contributions are computed in pQCD.

Lattice QCD – Example of correlation function $C(t)$ (RBC/UKQCD18)



Large discretization errors at short distance, large finite-volume errors and statistical errors at large distance

Window method (introduced in RBC/UKQCD 2018)

We therefore also consider a window method. Following Meyer-Bernecker 2011 and smearing over t to define the continuum limit we write

$$a_\mu = a_\mu^{\rm SD} + a_\mu^{\rm W} + a_\mu^{\rm LD}$$

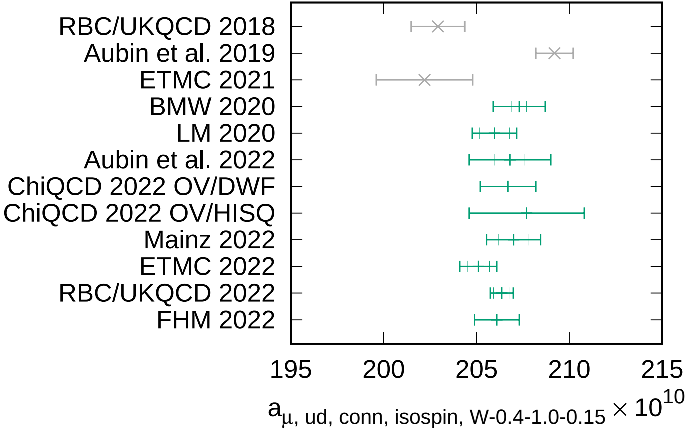
with

$$\begin{aligned}
a_\mu^{\rm SD} &= \sum_t C(t) w_t [1 - \Theta(t, t_0, \Delta)]\,, \\
a_\mu^{\rm W} &= \sum_t C(t) w_t [\Theta(t, t_0, \Delta) - \Theta(t, t_1, \Delta)]\,, \\
a_\mu^{\rm LD} &= \sum_t C(t) w_t \Theta(t, t_1, \Delta)\,, \\
\Theta(t, t', \Delta) &= [1 + \tanh[(t - t')/\Delta]]\,/2\,.
\end{aligned}$$

All contributions are well-defined individually and can be computed from lattice or R-ratio via $C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s}) R(s) s e^{-\sqrt{s}t}$ with $R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to {\rm had})$.

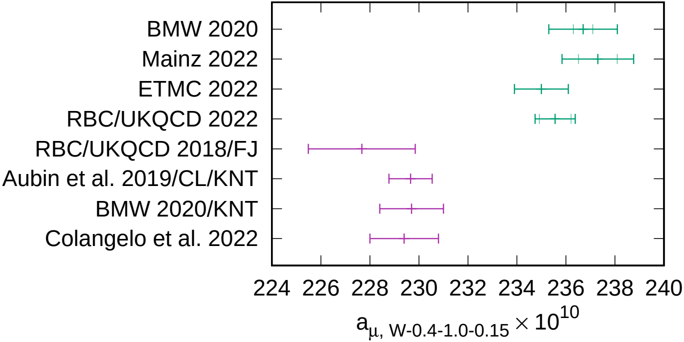
$a_\mu^{\rm W}$ has small statistical and systematic errors on lattice!

Use these windows as a lattice internal cross-check



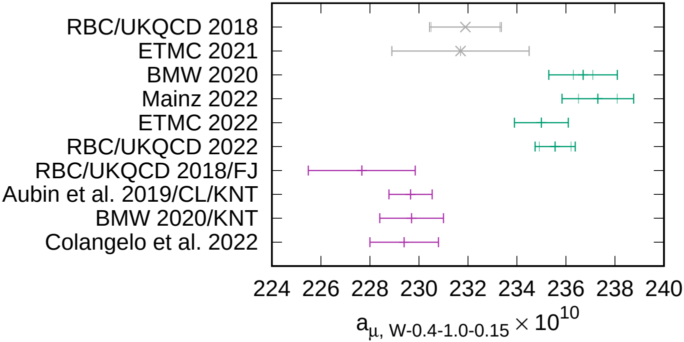
Isospin-symmetric light quark-connected contribution to a_μ^{W} for $t_0 = 0.4$ fm, $t_1 = 1.0$ fm; Note that the new RBC/UKQCD22 result was done in a fully blinded way with 5 independent analysis groups. It also uses 24 instead of 2 data points for the continuum extrapolation compared to the pioneering RBC/UKQCD18 result with which it is in 2.1σ tension.

Use these windows as a lattice internal cross-check



Isospin-symmetric light quark-connected contribution to a_μ^{SD} for $t_0 =$ fm; consistent with pQCD (RBC/UKQCD 2022)

Use these windows as a lattice internal cross-check



Multiple complete lattice QCD results for a_μ^{W} for $t_0 = 0.4$ fm, $t_1 = 1.0$ fm now also exist that exhibit a tension with the R-ratio of approximately 3.6σ.

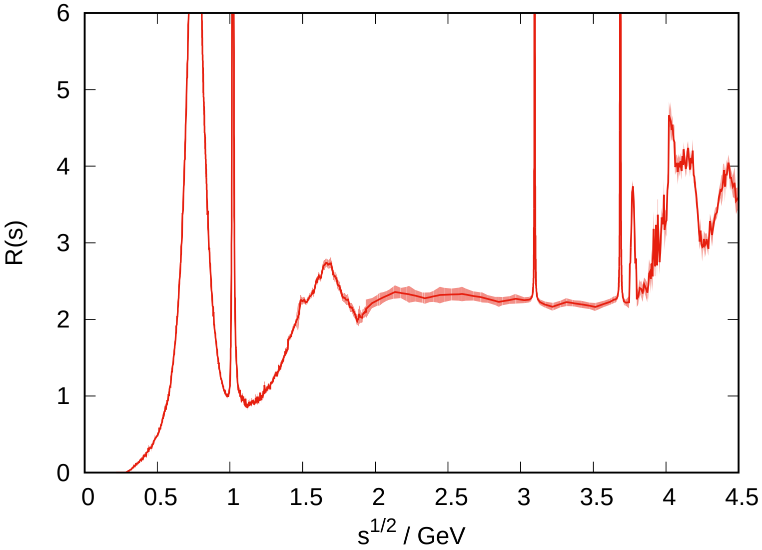
Summary of current status

- Short distance window (up to $t_0 = 0.4$ fm) dominated by pQCD, no sign of tension between data-driven (+pQCD) and LQCD
- Intermediate window ($t_0 = 0.4$ fm, $t_1 = 1.0$ fm), we have now established a 3.6σ tension of

$$a_{\mathrm{W}}^{\mathrm{Lattice}} - a_{\mathrm{W}}^{\mathrm{Data-Driven}} = 6.2(1.7) \times 10^{-10}\,.$$

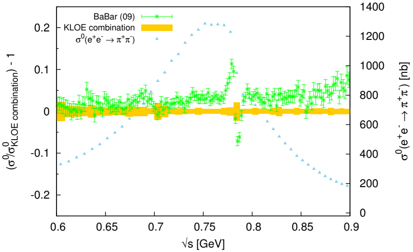
- The long-distance window is at this point not yet independently checked!
- The total a_μ BMW20 result lies approximately 15×10^{-10} above data-driven results.

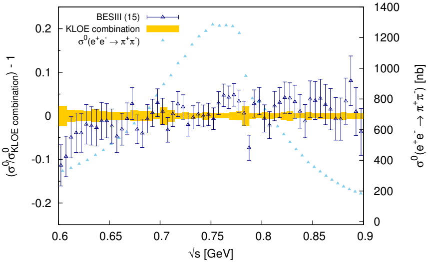
Consistency of lattice result with R-ratio

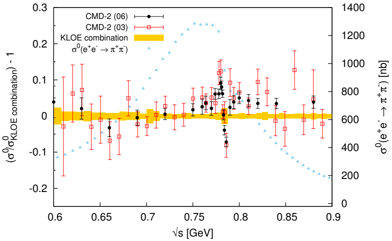


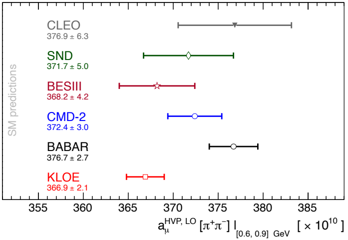
$$R(s) = \frac{3s}{4\pi\alpha^2}\sigma(s, e^+e^- \to \mathrm{had})\,, \quad C(t) = \frac{1}{12\pi^2}\int_0^\infty d(\sqrt{s})R(s)se^{-\sqrt{s}t}$$

Tensions in input data, however, already taken into account in WP20 merger of KNT19 and DHMZ19:









What does tension in windows mean for R-ratio?

If there is a shift in R-ratio, it crucially depends on which energy to understand what the impact on $\Delta\alpha$ and EW precision physics is.

Express Euclidean Windows in time-like region:

$$a_\mu = \int_0^\infty ds\, R(s) K(s) \tag{1}$$

and window

$$a_\mu^{\mathrm{W}} = \int_0^\infty ds\, R(s) K(s) P(s)\,. \tag{2}$$

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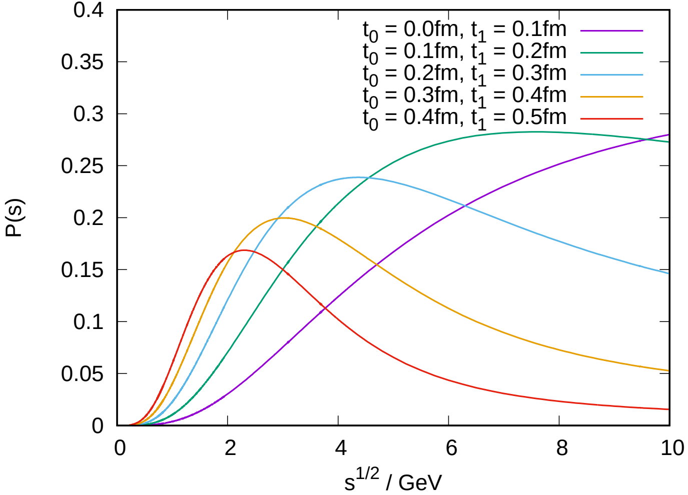
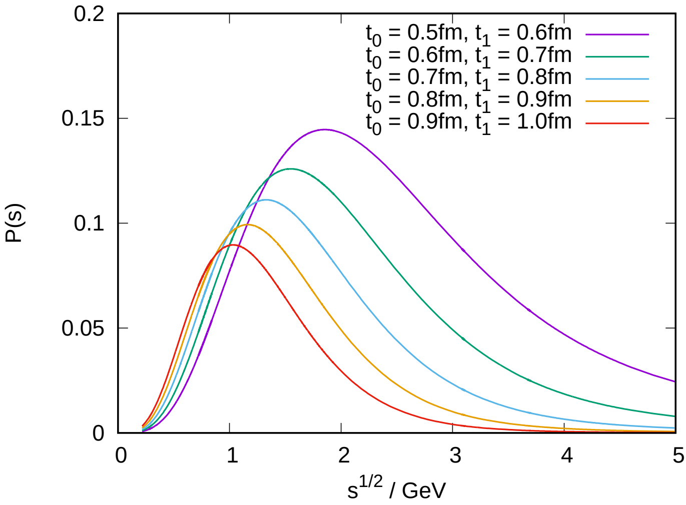
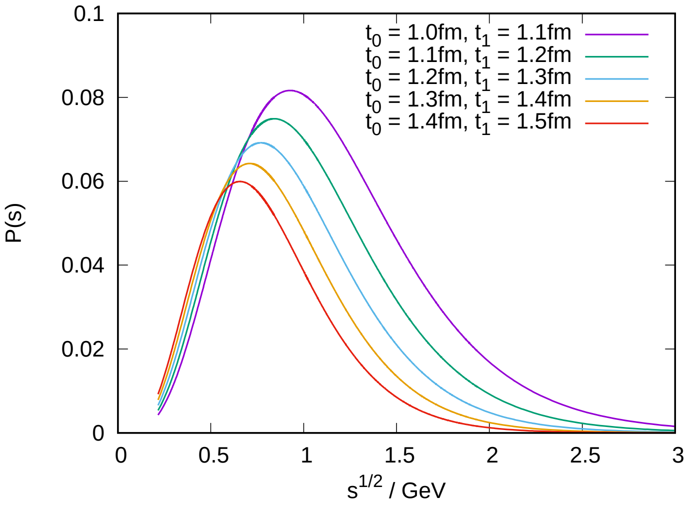


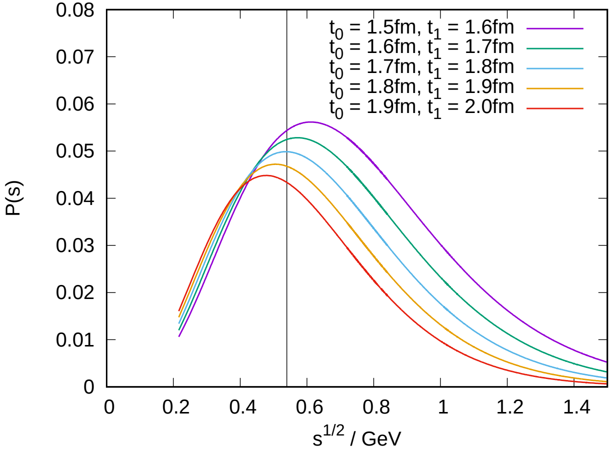
Study of windows for different t_0 and t_1 can give some energy resolution!



Study of windows for different t_0 and t_1 can give some energy resolution!

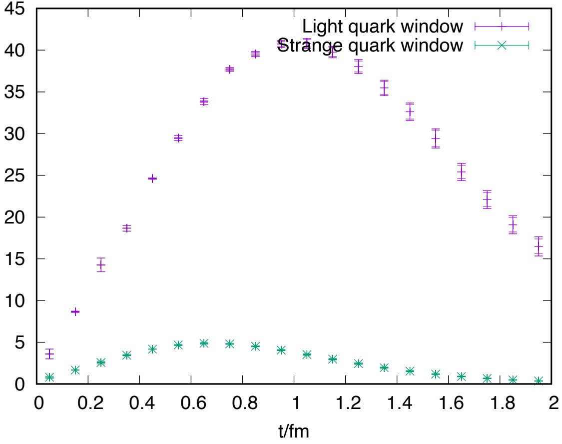


Study of windows for different t_0 and t_1 can give some energy resolution!



Below black line, we can use Lellouche-Lüscher-Meyer formalism to get $R(s)$ from lattice directly! Programs for this by Mainz and RBC/UKQCD.

First results for more windows already available - Lehner & Meyer 2020



Here: $t_0 = t,\ t_1 = t + 0.1\text{fm}$

No results for QED, SIB, and charm contribution yet available.

First results for more windows already available - Lehner & Meyer 2020

t_0/fm	t_1/fm	Δ/fm	$a_\mu^{\rm ud,conn.,isospin}10^{10}$	$a_\mu^{\rm s,conn.,isospin}10^{10}$
Total			657(26)(12)	52.83(22)(65)
0.0	0.1	0.15	3.60(00)(59)	0.81(00)(12)
0.1	0.2	0.15	8.649(03)(73)	1.666(01)(12)
0.2	0.3	0.15	14.27(01)(82)	2.57(00)(16)
0.3	0.4	0.15	18.67(02)(35)	3.448(05)(65)
0.4	0.5	0.15	24.617(35)(63)	4.170(07)(20)
0.5	0.6	0.15	29.47(06)(29)	4.666(10)(59)
0.6	0.7	0.15	33.85(10)(37)	4.866(13)(74)
0.7	0.8	0.15	37.71(14)(15)	4.799(16)(39)
0.8	0.9	0.15	39.55(20)(21)	4.505(17)(44)
0.9	1.0	0.15	40.77(27)(31)	4.058(19)(65)
1.0	1.1	0.15	40.86(44)(41)	3.527(19)(76)
1.1	1.2	0.15	39.81(54)(42)	2.973(19)(75)
1.2	1.3	0.15	38.10(65)(51)	2.441(18)(77)
1.3	1.4	0.15	35.54(77)(53)	1.955(17)(67)
1.4	1.5	0.15	32.70(88)(56)	1.534(15)(60)
1.5	1.6	0.15	29.50(100)(58)	1.181(13)(52)
1.6	1.7	0.15	25.51(81)(66)	0.894(12)(44)
1.7	1.8	0.15	22.20(85)(66)	0.667(10)(37)
1.8	1.9	0.15	19.18(86)(67)	0.491(08)(30)
1.9	2.0	0.15	16.59(89)(75)	0.357(07)(24)
0.0	0.2	0.15	12.25(00)(52)	2.48(00)(11)
0.2	0.4	0.15	32.95(03)(48)	6.02(01)(10)
0.4	0.6	0.15	54.08(10)(29)	8.837(18)(74)
0.6	0.8	0.15	71.55(24)(38)	9.666(29)(91)
0.8	1.0	0.15	80.33(47)(44)	8.56(04)(10)
0.3	1.0	0.15	224.6(0.8)(1.1)	30.51(08)(25)
0.3	1.3	0.15	343.1(2.6)(2.0)	39.45(13)(35)
0.3	1.6	0.15	441.0(5.1)(3.4)	44.12(17)(49)
0.4	1.0	0.15	205.97(79)(90)	27.06(08)(21)
0.4	1.3	0.15	324.6(2.6)(1.9)	36.01(13)(36)
0.4	1.6	0.15	422.4(5.1)(3.5)	40.68(17)(51)
0.4	1.0	0.05	216.5(0.8)(6.2)	27.9(0.1)(1.1)
0.4	1.0	0.1	209.80(77)(79)	27.70(08)(21)
0.4	1.0	0.2	202.10(82)(91)	26.24(08)(21)

More results expected by other collaborations soon! See also one-sided windows computed in FHM2022a.

Summary

- In the intermediate window a 3.6σ tension between data-driven and lattice QCD is now established. This accounts for a shift of $O(6 \times 10^{-10})$.
- The difference between the total BMW 20 and the data-driven result is $O(15 \times 10^{-10})$.
- The study of multiple window quantities may give insight into the energy region driving such a tension.
- Over the next year, we may expect at least one additional complete LQCD calculation at the sub-percent level.