LFV theory talk

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Workshop on Muon Precision Physics

Where to look for New Physics at low-energy?

- **Processes very suppressed or even forbidden in the SM**
  - LFV processes ($\mu \to e\gamma$, $\mu \to e$ in N, $\tau \to \mu\gamma$, $\tau \to 3\mu$, $\cdots$)
  - CPV effects in the leptonic (e, μ) and neutron EDMs
  - FCNC & CPV in $B_{s,d}$ & D decay/mixing amplitudes

- **Processes predicted with high precision in the SM**
  - EWPO as $(g-2)_\mu$: $\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (2.51 \pm 0.59) \times 10^{-9}$ (4.2σ discrepancy!)
  - LFUV in $M \to \ell\nu$ (with $M = \pi, K, B$), $B \to D^{(*)}\ell\nu$, $B \to K\ell\ell'$, τ and Z decays

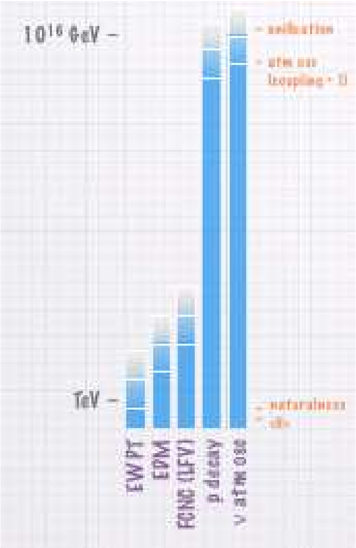
- **High-intensity frontier: A collective effort to determine the NP symmetries**
- **High-energy frontier: A unique effort to determine the NP scale**

- **Gravity** $\Longrightarrow$ $\Lambda_{\textbf{Planck}} \sim 10^{18-19}$ GeV
- **Neutrino masses** $\Longrightarrow$ $\Lambda_{\textbf{see-saw}} \lesssim 10^{15}$ GeV
- **BAU**: evidence of CPV beyond SM
  - Electroweak Baryogenesis $\Longrightarrow$ $\Lambda_{\textbf{NP}} \lesssim$ TeV
  - Leptogenesis $\Longrightarrow$ $\Lambda_{\textbf{see-saw}} \lesssim 10^{15}$ GeV
- **Hierarchy problem**: $\Longrightarrow$ $\Lambda_{\textbf{NP}} \lesssim$ TeV
- **Dark Matter (WIMP)** $\Longrightarrow$ $\Lambda_{\textbf{NP}} \lesssim$ TeV

SM = effective theory at the EW scale

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \sum_{d\geq 5} \frac{c_{ij}^{(d)}}{\Lambda_{NP}^{d-4}}\, O_{ij}^{(d)}$$

- $\mathcal{L}_{\rm eff}^{d=5} = \frac{y_\nu^{ij}}{\Lambda_{\rm see-saw}} L_i L_j \phi\phi$,
- $\mathcal{L}_{\rm eff}^{d=6}$ generates FCNC operators



$$\textbf{BR}(\ell_i \to \ell_j\gamma) \sim \frac{G_F^{-2}}{\Lambda_{NP}^4}$$

Experimental status

Process	Present	Experiment	Future	Experiment
$\mu \to e\gamma$	4.2×10^{-13}	MEG	$\approx 6 \times 10^{-14}$	MEG II
$\mu \to 3e$	1.0×10^{-12}	SINDRUM	$\approx 10^{-16}$	Mu3e
μ^- Au $\to e^-$ Au	7.0×10^{-13}	SINDRUM II	?	
μ^- Ti $\to e^-$ Ti	4.3×10^{-12}	SINDRUM II	?	
μ^- Al $\to e^-$ Al	–		$\approx 10^{-16}$	COMET, MU2e
$\tau \to e\gamma$	3.3×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to \mu\gamma$	4.4×10^{-8}	Belle & BaBar	$\sim 10^{-9}$	Belle II
$\tau \to 3e$	2.7×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$\tau \to 3\mu$	2.1×10^{-8}	Belle & BaBar	$\sim 10^{-10}$	Belle II
$d_e(\mathrm{e\ cm})$	1.1×10^{-29}	ACME	$\sim 3 \times 10^{-31}$	ACME III
$d_\mu(\mathrm{e\ cm})$	1.8×10^{-19}	Muon (g-2)	$\sim 10^{-22}$	PSI

Table: Present and future experimental sensitivities for relevant low-energy observables.

- So far, only upper bounds. Still excellent prospects for exp. improvements.
- We can expect a NP signal in all above observables below the current bounds.

On the muon g-2 puzzle

- **Status of $a_\mu \equiv \frac{g_\mu - 2}{2}$ as of April 7$^{\rm th}$ 2021 (with $a_\mu^{\rm SM}$ based on $a^{\rm HLO}_{\mu,\, e^+e^-}$)**

$$a_\mu^{\rm EXP} = 116592061(41) \times 10^{-11} \quad \text{[BNL + FNAL]}$$
$$a_\mu^{\rm SM} = 116591810(43) \times 10^{-11} \quad \text{[WP20]}$$

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = 251\,(59) \times 10^{-11} \qquad (4.2\sigma \text{ discrepancy!})$$

$$\underbrace{(0.1)_{\rm QED}\,,\quad (1)_{\rm EW}\,,\quad (18)_{\rm HLbL}\,,\quad (40)_{\rm HVP}\,,}_{(43)_{\rm TH}}\quad (41)_{\delta a_\mu^{\rm EXP}}\,.$$

  - Hadronic uncertainties (HLbL & HVP) are very hard to improve.
  - $\delta a_\mu^{\rm EXP} \approx 16 \times 10^{-11}$ by the E989 Muon $g-2$ exp. in a few years.

- **Low-energy determinations of Δa_μ assume that systematic and hadronic uncertainties are under control at the outstanding level of $\Delta a_\mu < 10^{-9}$!**

New Physics for the muon $g-2$: at which scale?

- **Δa_μ discrepancy at $\sim 4.2\,\sigma$ level:**

$$\Delta a_\mu = a_\mu^{\rm EXP} - a_\mu^{\rm SM} \equiv a_\mu^{\rm NP} = (2.51 \pm 0.59) \times 10^{-9}$$

$$\Delta a_\mu \equiv a_\mu^{\rm NP} \approx (a_\mu^{\rm SM})_{weak} \approx \frac{m_\mu^2}{16\pi^2 v^2} \approx 2 \times 10^{-9}$$

  - NP is at the weak scale ($\Lambda \approx v$) and weakly coupled to SM particles.*
  - NP is very light ($\Lambda \lesssim 1$ GeV) and feebly coupled to SM particles.
  - NP is very heavy ($\Lambda \gg v$) and strongly coupled to SM particles.

*Favoured by the *hierarchy problem* and by a WIMP DM candidate but disfavoured by the LEP and LHC bounds (supersymmetry being the most prominent example).

On leptonic dipoles: $\ell \to \ell' \gamma$

- **NP effects are encoded in the effective Lagrangian**

$$\mathcal{L} = e \frac{m_\ell}{2} \left(\bar{\ell}_R \sigma_{\mu\nu} A_{\ell\ell'} \ell'_L + \bar{\ell}'_L \sigma_{\mu\nu} A^\star_{\ell\ell'} \ell_R \right) F^{\mu\nu} \qquad \ell, \ell' = e, \mu, \tau \,,$$

  - **Branching ratios of $\ell \to \ell' \gamma$**

$$\frac{\mathrm{BR}(\ell \to \ell' \gamma)}{\mathrm{BR}(\ell \to \ell' \nu_\ell \bar{\nu}_{\ell'})} = \frac{48 \pi^3 \alpha}{G_F^2} \left(|A_{\ell\ell'}|^2 + |A_{\ell'\ell}|^2 \right) .$$

  - **Δa_ℓ and leptonic EDMs**

$$\Delta a_\ell = 2 m_\ell^2 \, \mathrm{Re}(A_{\ell\ell}), \qquad \frac{d_\ell}{e} = m_\ell \, \mathrm{Im}(A_{\ell\ell}) \,.$$

  - **“Naive scaling”:** a broad class of NP theories contributes to Δa_ℓ and d_ℓ as

$$\frac{\Delta a_\ell}{\Delta a_{\ell'}} = \frac{m_\ell^2}{m_{\ell'}^2}, \qquad \frac{d_\ell}{d_{\ell'}} = \frac{m_\ell}{m_{\ell'}} \,.$$

Model-independent predictions

- $\mathrm{BR}(\ell_i \to \ell_j \gamma)$ **vs.** $(g-2)_\mu$

$$\mathrm{BR}(\mu \to e\gamma) \approx 3 \times 10^{-13} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{e\mu}}{10^{-5}}\right)^2$$

$$\mathrm{BR}(\tau \to \mu\gamma) \approx 4 \times 10^{-8} \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right)^2 \left(\frac{\theta_{\mu\tau}}{10^{-2}}\right)^2$$

- **EDMs vs.** $(g-2)_\mu$

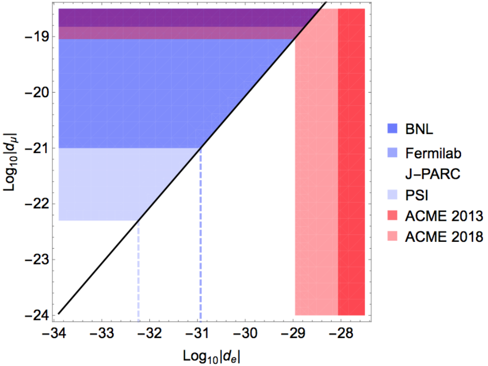
$$d_e \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 10^{-29} \left(\frac{\phi_e^{CPV}}{10^{-5}}\right) \; e\,\mathrm{cm}\,,$$

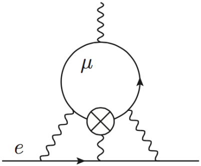
$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3 \times 10^{-9}}\right) 2 \times 10^{-22}\, \phi_\mu^{CPV} \; e\,\mathrm{cm}\,,$$

- **Main messages:**
  - $\Delta a_\mu \approx (3 \pm 1) \times 10^{-9}$ **requires a nearly flavor and CP conserving NP**
  - **Large effects in the muon EDM** $d_\mu \sim 10^{-22}\, e\,\mathrm{cm}$ **are still allowed!**

[Giudice, P.P., & Passera, '12]

Experimental status of the muon EDM





$$d_\mu \leq 10^{-21}\, e\,\mathrm{cm}\left(\frac{d_e}{10^{-31}\, e\,\mathrm{cm}}\right)$$

[Crivellin, Hoferichter & Schmidt-Wellenburg, '18]

$$d_\mu \simeq \left(\frac{\Delta a_\mu}{3\times 10^{-9}}\right) 2\times 10^{-22}\,\phi_\mu^{CPV}\; e\,\mathrm{cm}\,,$$

[Giudice, PP & Passera, '12]

Not only $\mu \to e\gamma$...

- **LFV** operators **@ dim-6**

$$\mathcal{L}_{\rm eff} = \mathcal{L}_{\rm SM} + \frac{1}{\Lambda^2_{\bf LFV}} \mathcal{O}^{\bf dim-6} + \dots .$$

$$\mathcal{O}^{\rm dim-6} \ni \;\; \bar{\mu}_R \, \sigma^{\mu\nu} \, H \, e_L \, F_{\mu\nu} \, , \;\; (\bar{\mu}_L \gamma^\mu e_L) \left(\bar{f}_L \gamma^\mu f_L\right) \, , \;\; (\bar{\mu}_R e_L) \left(\bar{f}_R f_L\right) \, , \;\; f = e, u, d$$

- $\ell \to \ell' \gamma$ probe ONLY the dipole-operator (at tree level)
- $\ell_i \to \ell_j \bar{\ell}_k \ell_k$ and $\mu \to e$ in Nuclei probe dipole and 4-fermion operators
- When the dipole-operator is dominant:

$$\mathrm{BR}(\ell_i \to \ell_j \ell_k \bar{\ell}_k) \approx \alpha \times \mathrm{BR}(\ell_i \to \ell_j \gamma)$$
$$\mathrm{CR}(\mu \to e \text{ in N}) \approx \alpha \times \mathrm{BR}(\mu \to e\gamma)$$

$$\frac{\mathrm{BR}(\mu \to \mathbf{3e})}{\mathbf{3 \times 10^{-15}}} \approx \frac{\mathrm{BR}(\mu \to \mathbf{e}\gamma)}{\mathbf{5 \times 10^{-13}}} \approx \frac{\mathrm{CR}(\mu \to \mathbf{e} \text{ in N})}{\mathbf{3 \times 10^{-15}}}$$

- **Ratios like** $Br(\mu \to e\gamma)/Br(\tau \to \mu\gamma)$ **probe the NP flavor structure**
- **Ratios like** $Br(\mu \to e\gamma)/Br(\mu \to eee)$ **probe the NP operator at work**

Hints of LFUV in semileptonic B decays

Coeff.	best fit	1σ	pull
C_9^μ	-1.56	$[-2.87, -0.71]$	4.1σ
C_{10}^μ	$+1.20$	$[+0.58, +2.00]$	4.2σ
C_9^e	$+1.54$	$[+0.76, +2.48]$	4.3σ
C_{10}^e	-1.27	$[-2.08, -0.61]$	4.3σ
$C_9^\mu = -C_{10}^\mu$	-0.63	$[-0.98, -0.32]$	4.2σ
$C_9^e = -C_{10}^e$	$+0.76$	$[+0.36, +1.27]$	4.3σ
$C_9^e = C_{10}^e$	-1.91	$[-2.71, -1.10]$	3.9σ
$C_9^{\prime\,\mu}$	-0.05	$[-0.57, +0.46]$	0.2σ
$C_{10}^{\prime\,\mu}$	$+0.03$	$[-0.44, +0.51]$	0.1σ
$C_9^{\prime\,e}$	$+0.07$	$[-0.49, +0.69]$	0.2σ
$C_{10}^{\prime\,e}$	-0.04	$[-0.57, +0.45]$	0.2σ

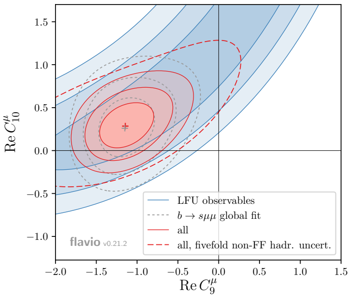
$$O_9^\ell = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \ell)$$

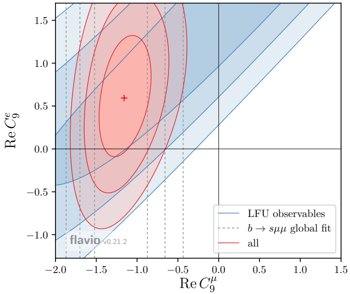
$$O_9^{\prime\,\ell} = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \ell)$$

$$O_{10}^\ell = (\bar{s}\gamma_\mu P_L b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

$$O_{10}^{\prime\,\ell} = (\bar{s}\gamma_\mu P_R b)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

[Altmannshofer, Stangl, & Straub, '17]





High-energy effective Lagrangian

- **Global fits** of $B \to K^* \ell\ell$ data favour (not exclusively) an effective 4-fermion operator involving left-handed currents $(\bar{s}_L \gamma_\mu b_L)(\bar{\mu}_L \gamma_\mu \mu_L)$, i.e. the $C_9 = -C_{10}$ solution [Hiller et al., '14, Hurth et al.,'14, Altmannshofer and Straub '14, Descotes-Genon et al., '15, ...].

- **A simultaneous explanation of both $R_K^{\mu/e}$ and $R_D^{\tau/\ell}$ anomalies naturally selects a left-handed operator $(\bar{c}_L \gamma_\mu b_L)(\bar{\tau}_L \gamma_\mu \nu_L)$ which is related to $(\bar{s}_L \gamma_\mu b_L)(\bar{\mu}_L \gamma_\mu \mu_L)$ by the $SU(2)_L$ gauge symmetry** [Bhattacharya et al., '14].

- **This picture can work only if NP couples much more strongly to the third generation than to the first two.** Two interesting scenarios are:
  - **Lepton Flavour Violating case:** NP couples in the interaction basis only to third generations. Couplings to lighter generations are generated by the misalignment between the mass and the interaction bases [Glashow, Guadagnoli and Lane, '14] .
  - **Lepton Flavour Conserving case:** NP couples dominantly to third generations but LFV does not arise if the groups $U(1)_e \times U(1)_\mu \times U(1)_\tau$ are unbroken [Alonso et al., '15].

LFV case: high-energy effective Lagrangian

- In the energy window between the EW scale v and the NP scale Λ, NP effects are described by $\mathcal{L}=\mathcal{L}_{\rm SM}+\mathcal{L}_{\rm NP}$ with $\mathcal{L}$ invariant under $SU(2)_L \otimes U(1)_Y$.

$$\mathcal{L}_{\rm NP} = \frac{C_1}{\Lambda^2}\left(\bar{q}_{3L}\gamma^\mu q_{3L}\right)\left(\bar{\ell}_{3L}\gamma_\mu \ell_{3L}\right) + \frac{C_3}{\Lambda^2}\left(\bar{q}_{3L}\gamma^\mu \tau^a q_{3L}\right)\left(\bar{\ell}_{3L}\gamma_\mu \tau^a \ell_{3L}\right).$$

- After EWSB we move to the mass basis through the unitary transformations

$$u_L \to V_u u_L \qquad d_L \to V_d d_L \qquad \nu_L \to U_e \nu_L \qquad e_L \to U_e e_L\,,$$

$$\begin{aligned}\mathcal{L}_{\rm NP} = \frac{1}{\Lambda^2}[&(C_1+C_3)\,\lambda^d_{ij}\lambda^e_{kl}\,(\bar{d}_{Li}\gamma^\mu d_{Lj})(\bar{e}_{Lk}\gamma_\mu e_{Ll}) \; + && B\to K^{(*)}\ell\ell' \\ &(C_1-C_3)\,\lambda^d_{ij}\lambda^e_{kl}\,(\bar{d}_{Li}\gamma^\mu d_{Lj})(\bar{\nu}_{Lk}\gamma_\mu \nu_{Ll})] \; + && B\to K^{(*)}\nu\nu \\ &2C_3(V\lambda^d)_{ij}\lambda^e_{kl}\,(\bar{u}_{Li}\gamma^\mu d_{Lj})(\bar{e}_{Lk}\gamma_\mu \nu_{Ll})+h.c.] && B\to D^{(*)}\ell\nu\end{aligned}$$

[Calibbi, Crivellin, Ota, '15]

$$\lambda^d_{ij} = V^*_{d3i}V_{d3j} \qquad \lambda^e_{ij} = U^*_{e3i}U_{e3j} \qquad V_u^\dagger V_d = V_{\rm CKM} \equiv V$$

- Assumption for the flavor structure: $\lambda^{d,e}_{33}\approx 1$, $\lambda^{d,e}_{22} = |\lambda^{d,e}_{23}|^2$, $\lambda^{d,e}_{13}=0$.

Construction of the low-energy effective Lagrangian: running and matching

- We use the renormalization group equations (RGEs) to evolve the effective lagrangian $\mathcal{L}_{\rm NP}$ from $\mu \sim \Lambda$ down to $\mu \sim 1$ GeV. This is done is three steps:
  - First step: the RGEs in the unbroken $SU(2)_L \otimes U(1)_Y$ theory [Manohar et al.,'13] are used to compute the coefficients in the effective lagrangian down to a scale $\mu \sim m_Z$.
  - Second step: the coefficients are matched to those of an effective lagrangian for the theory in the broken symmetry phase of $SU(2)_L \otimes U(1)_Y$, that is $U(1)_{el}$.
  - Third step: the coefficients of this effective lagrangian are computed at $\mu \sim 1$ GeV using the RGEs for the theory with the only $U(1)_{el}$ gauge group.
- Then we take matrix elements of the relevant operators. The scale dependence of the RGE contributions cancels with that of the matrix elements.

[Feruglio, P.P., Pattori, PRL '16, '17]

Purely leptonic effective Lagrangian

- **Quantum effects generate a purely leptonic effective Lagrangian**:

$$\mathcal{L}_{\rm eff}^{\rm NC} = -\frac{4G_F}{\sqrt{2}}\lambda^e_{ij}\left[(\bar{e}_{Li}\gamma_\mu e_{Lj})\sum\nolimits_\psi \bar{\psi}\gamma^\mu\psi\,(2g^Z_\psi \mathbf{c}^{\mathbf{e}}_{\mathbf{t}} - Q_\psi \mathbf{c}^{\mathbf{e}}_{\gamma}) + h.c.\right]$$

$$\mathcal{L}_{\rm eff}^{\rm CC} = -\frac{4G_F}{\sqrt{2}}\lambda^e_{ij}\left[\mathbf{c}^{\mathbf{cc}}_{\mathbf{t}}(\bar{e}_{Li}\gamma_\mu \nu_{Lj})(\bar{\nu}_{Lk}\gamma^\mu e_{Lk} + \bar{u}_{Lk}\gamma^\mu V_{kl} d_{Ll}) + h.c.\right]$$

$\psi = \{\nu_{Lk}, e_{Lk,Rk}, u_{L,R}, d_{L,R}, s_{L,R}\}$ $\qquad$ $g^Z_\psi = T_3(\psi) - Q_\psi \sin^2\theta_W$

$$\mathbf{c}^{\mathbf{e}}_{\mathbf{t}} = \mathbf{y}^{\mathbf{2}}_{\mathbf{t}}\frac{3}{32\pi^2}\frac{v^2}{\Lambda^2}(C_1 - C_3)\lambda^u_{33}\log\frac{\Lambda^2}{m_t^2}$$

$$\mathbf{c}^{\mathbf{cc}}_{\mathbf{t}} = \mathbf{y}^{\mathbf{2}}_{\mathbf{t}}\frac{3}{16\pi^2}\frac{v^2}{\Lambda^2}C_3\,\lambda^u_{33}\log\frac{\Lambda^2}{m_t^2}$$

$$\mathbf{c}^{\mathbf{e}}_{\gamma} = \frac{\mathbf{e}^{\mathbf{2}}}{48\pi^2}\frac{v^2}{\Lambda^2}\left[(3C_3 - C_1)\log\frac{\Lambda^2}{\mu^2} + \ldots\right]$$

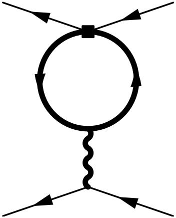


Figure: Diagram generating a four-lepton process.

- Top-quark yukawa interactions affect both neutral and charged currents.
- Gauge interactions are proportional to e^2 and to the e.m. current.

LFV decays

- **LFV B decays (tree-level)**

$$\mathcal{B}(B\to K\tau\mu) \approx 4\times 10^{-8}\left|C_9^{\mu\tau}\right|^2 \approx 10^{-7}\left|\frac{C_9^{\mu\mu}}{0.5}\right|^2\left|\frac{0.3}{\lambda_{23}^e}\right|^2 ,$$

- **LFV τ decays (1-loop)**

$$\mathcal{B}(\tau\to 3\mu) \approx 5\times 10^{-8}\,\frac{(C_1 - C_3)^2}{\Lambda^4(\mathrm{TeV})}\left(\frac{\lambda_{23}^e}{0.3}\right)^2$$

$$\mathcal{B}(\tau\to 3\mu) \approx \mathcal{B}(\tau\to\mu\rho) \approx \mathcal{B}(\tau\to\mu\pi)$$

- **Experimental bounds** [HFAG]:

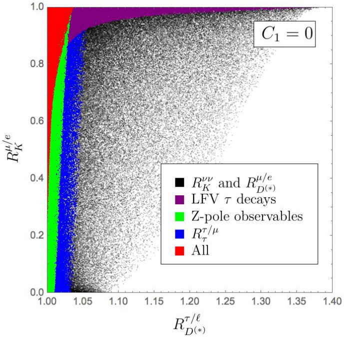
$$\mathcal{B}(B\to K\tau\mu)_{\rm exp} \leq 4.8\times 10^{-5}$$

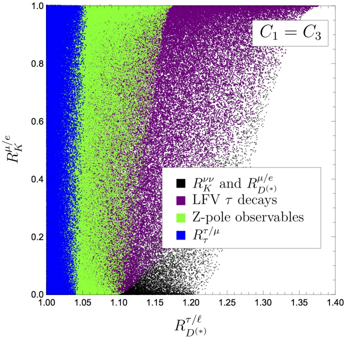
$$\mathcal{B}(\tau\to 3\mu)_{\rm exp} \leq 2.1\times 10^{-8}$$

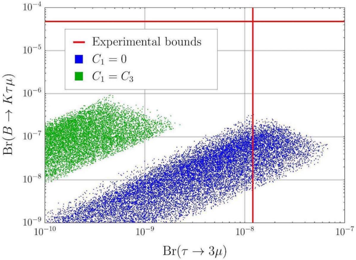
$$\mathcal{B}(\tau\to \mu\rho)_{\rm exp} \leq 1.2\times 10^{-8}$$

$$\mathcal{B}(\tau\to \mu\pi)_{\rm exp} \leq 2.7\times 10^{-8}$$

B anomalies







[Feruglio, P.P., Pattori, PRL '16, '17]

Conclusions and future prospects

- **Important questions in view of ongoing/future experiments are**:
  - What are the expected deviations from the SM predictions induced by TeV NP?
  - Which observables are not limited by theoretical uncertainties?
  - In which case we can expect a substantial improvement on the experimental side?
  - What will the measurements teach us if deviations from the SM are [not] seen?

- **(Personal) answers**:
  - We can expect any deviation from the SM expectations below the current bounds.
  - LFV processes, leptonic EDMs and LFUV observables do not suffer from theoretical limitations and there are still excellent prospects for experimental improvements.
  - If the muon $g-2$ anomaly will survive, we expect relevant enhancements in leptonic EDMs (especially in the muon EDM) and LFV decays $\mu \to e\gamma$, $\mu \to eee$, ...
  - The observed LFUV in $B \to D^{(*)}\ell\nu$, $B \to K\ell\ell'$ might be true NP signals. If confirmed, it is worth to look for LFV in $B \to K\tau\mu$, $\tau \to \mu\ell\ell$, $\tau \to \mu\rho$, ...

Message: an exciting Physics program is in progress at the Intensity Frontier!